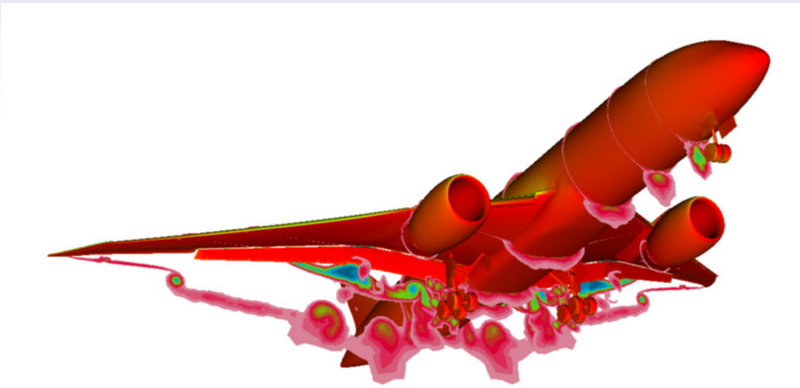
Mathematical Modeling
Part II



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Agenda

- Background
- Computational/Predictive Methods
- Wind-Tunnel Testing
- Flight-Testing
- Summary

Background

- Modeling the flight dynamics of a fixed- or rotary wing vehicle requires a mathematical model that describes how it flies and is controlled through the air and handles on ground.
  - From Newton's second law

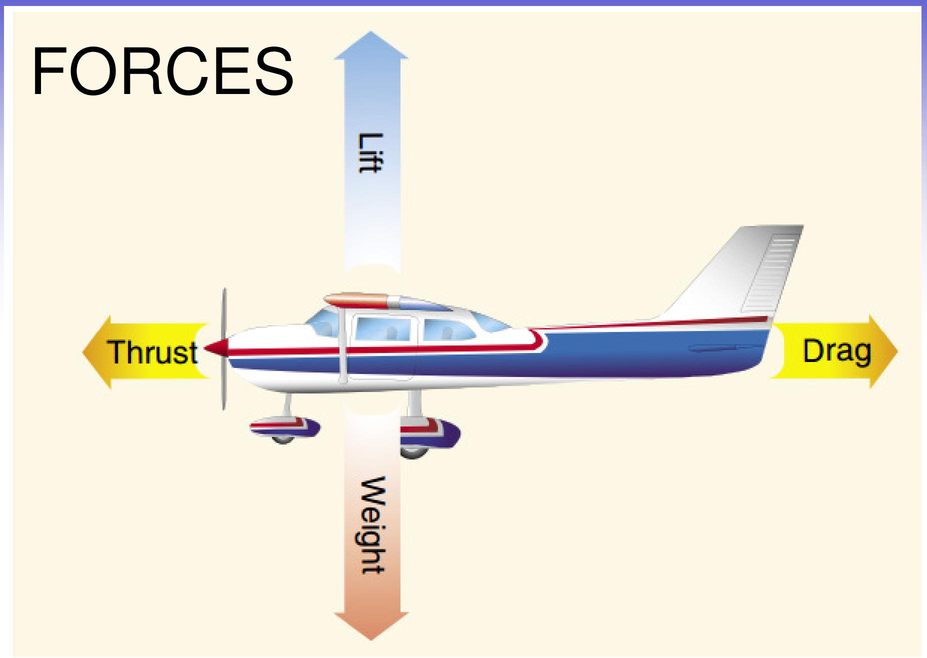
$$\dot{\boldsymbol{v}}_{\boldsymbol{B}} = \frac{1}{m}\boldsymbol{F}_{\boldsymbol{B}}\,(\boldsymbol{\omega}_{\boldsymbol{B}} \times \boldsymbol{v}_{\boldsymbol{B}}) + \boldsymbol{B}\boldsymbol{g}_{\boldsymbol{o}}$$

$$\dot{\boldsymbol{\omega}}_{\boldsymbol{B}} = -\boldsymbol{J}^{-1}\big(\boldsymbol{\omega}_B \times (\boldsymbol{J}\boldsymbol{\omega}_B)\big) + \boldsymbol{J}^{-1}\boldsymbol{T}_{\boldsymbol{B}}$$

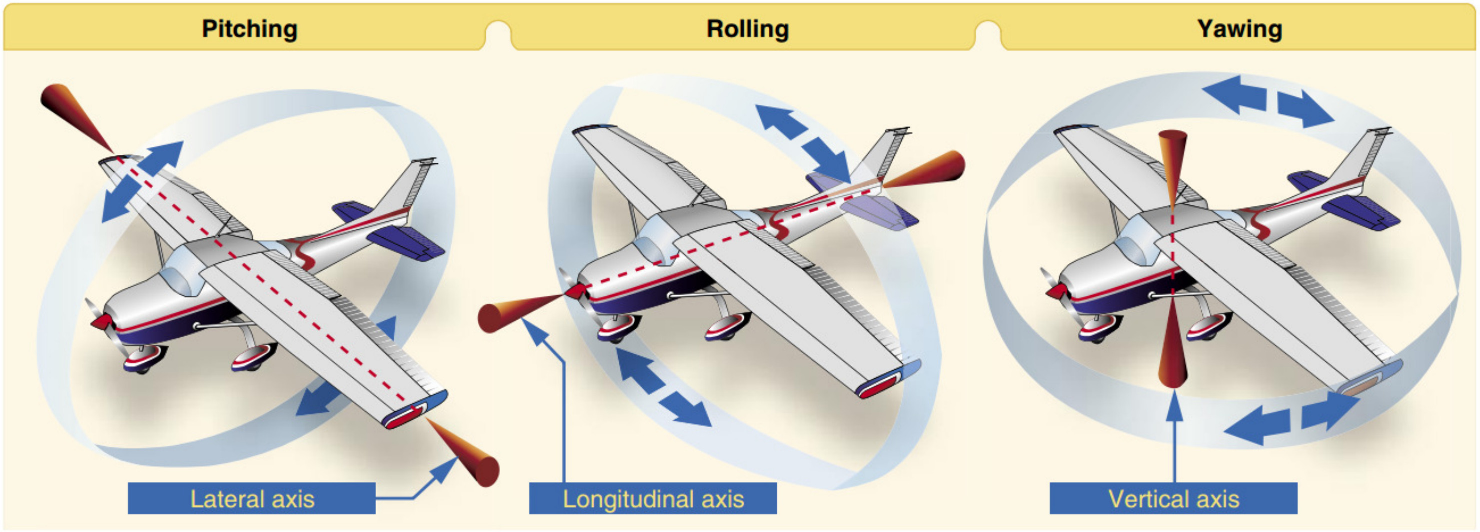
Forces

Moments

  - We need to determine $\boldsymbol{F}_{\boldsymbol{B}}$ and $\boldsymbol{T}_{\boldsymbol{B}}$



MOMENTS



Images from Pilot's Handbook of Aeronautical Knowledge (FAA)

Breakdown

- Forces and moments acting on the vehicle
  - Propulsion
  - Ground Reaction
  - Aerodynamic
    - Stability
    - Control

TODAY's FOCUS

Where do these data come from?

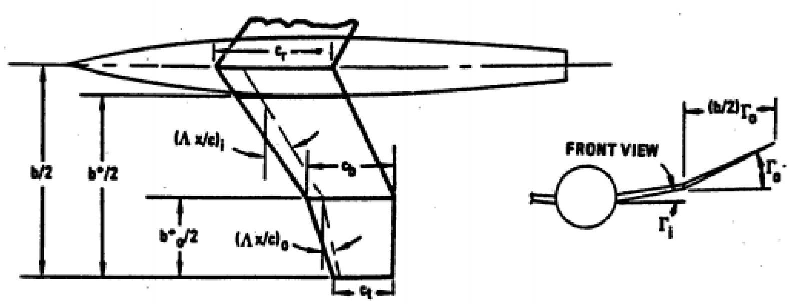
Computational/Predictive Methods

- Theoretical and or semi-empirical algorithms and data to compute aerodynamic forces and moments

Computational/Predictive Methods

- USAF DATCOM
  - Example of semi-empirical predictive method for fixed-wing vehicles
  - "Systematic summary of methods for estimating basics stability and control derivatives" (AFWAL-TR-83-3048)
  - Digital version implements methods and techniques and is available online
  - Input requires geometric information of vehicle

NAMELISTS WGPLNF, HTPLNF, VTPLNF, AND VFPLNF



Computational/Predictive Methods

- USAF DATCOM
  - Output is in the form of stability and control derivatives

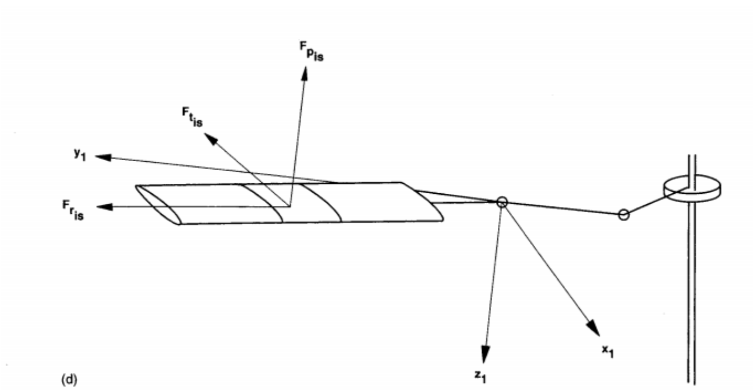
$$T_B = \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{Aero} + \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{Propulsion} + \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{Ground} + \cdots$$

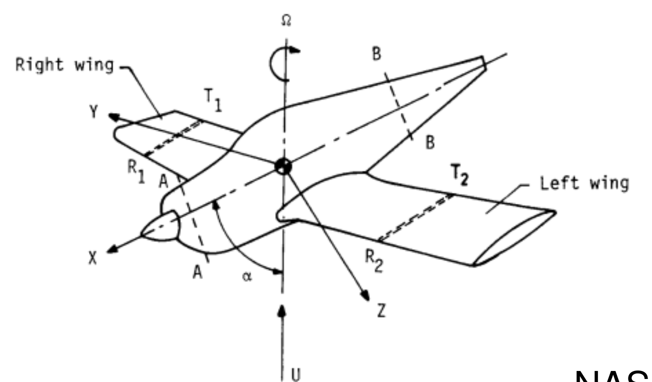
$$\frac{M_{Aero}}{QS\bar{c}} = C_{m_{Aero}} = (C_{m_o} + C_{m_\alpha}\alpha + C_{m_{de}}de + C_{m_q}\frac{q\bar{c}}{2V})$$

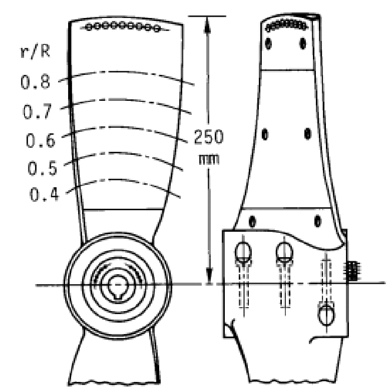
DATCOM Output

Computational/Predictive Methods

- Strip Methods/Blade Element
  - Computational approach that uses local flow state (Airspeed and angle of attack) to compute local forces for an strip/element on a vehicle, rotor, or propeller. Often uses 2D characteristics
  - Local forces and locations added to compute total forces and moments







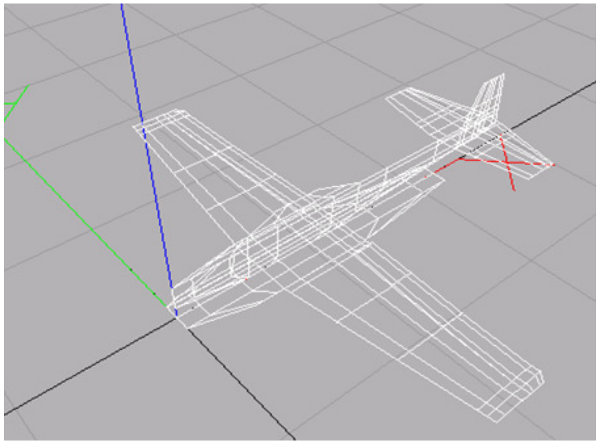
NASA TM4009

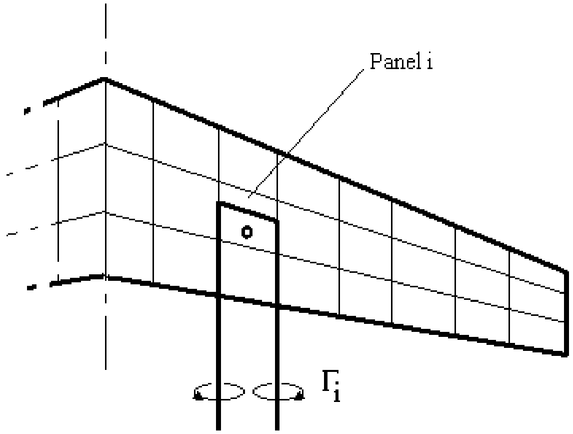
Computational/Predictive Methods

- Strip Methods/Blade Element
  - Output of these methods are total forces for the component being analyzed at a given geometric setting
  - Example Rotors:
    - $F_{Z_{Rotor}} = \sum_{i=0}^{N_{Rotors}} F_{i_{Rotor}}$
    - Where, $F_{i_{Rotor}} = \sum_{j=0}^{N_{Elements}} F_{j_{Elements}}$
  - Example Airplane
    - $F_{Z_{Total}} = F_{Z_{wing}} + F_{Z_{Fuselage}} + F_{Z_{Horizontal\,Tail}}$

Computational/Predictive Methods

- Low-Order Computational Aerodynamics
  - Vortex Lattice
    - Inviscid Solution – Does not compute account for friction and separation.
    - Uses a “simple” planar definition of the vehicle and camber information to compute the strength of horseshoe vortices placed at panel control points.



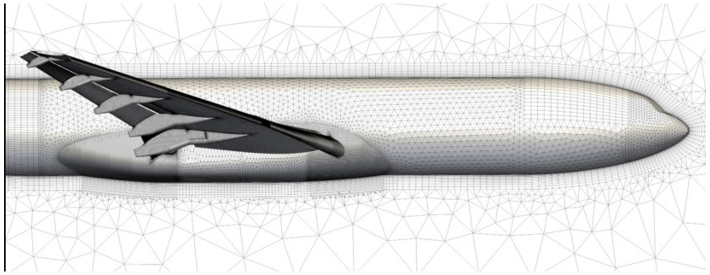


Computational/Predictive Methods

- Low-Order Computational Aerodynamics
  - Output of Low-Order computational aerodynamics methods is similar to strip methods.
    - Horseshoe vortices are used to compute forces on each panel which are summed for components and entire solutions.
    - Extraction of control effects require multiple solutions
    - Surface pressure distributions can be computed using methods like Vortex Lattice.

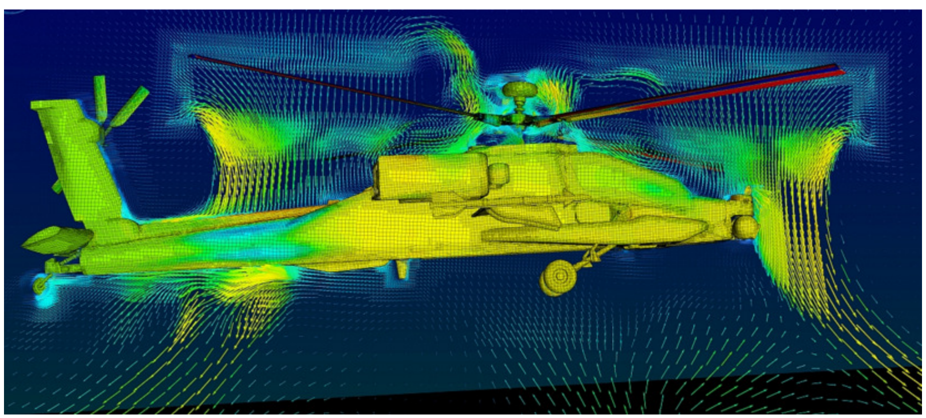
Computational/Predictive Methods

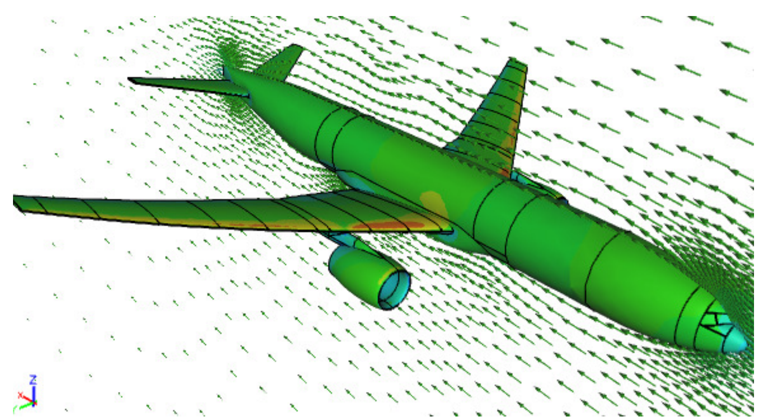
- Computational Fluid Dynamics (CFD)
  - High-Order computational solutions
    - CFD solutions solve the equations that describe the dynamics of a viscous fluid about a body and or through cavities. Allows modeling of separation, turbulence, boundary layer effects.
    - Requires very detailed geometric representation and computational grid
    - Computationally expensive and time consuming
      - Days for a single solution



Computational/Predictive Methods

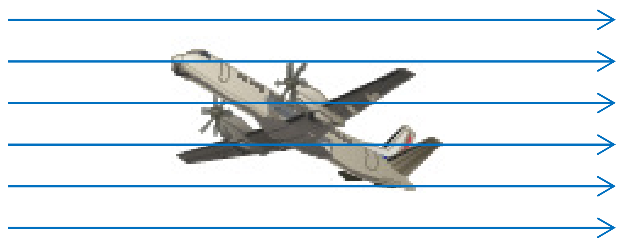
- Computational Fluid Dynamics
  - Output of CFD are total forces that can be used to resolve component based effects and moments.
  - CFD use for simulation development is growing, but not widely used because of cost.
  - CFD can provide Off-Body flow fields

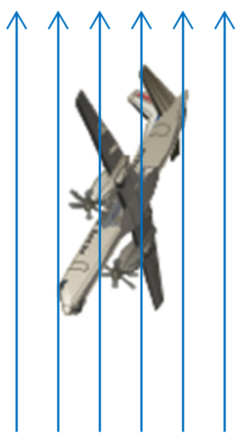




Wind-Tunnel Testing

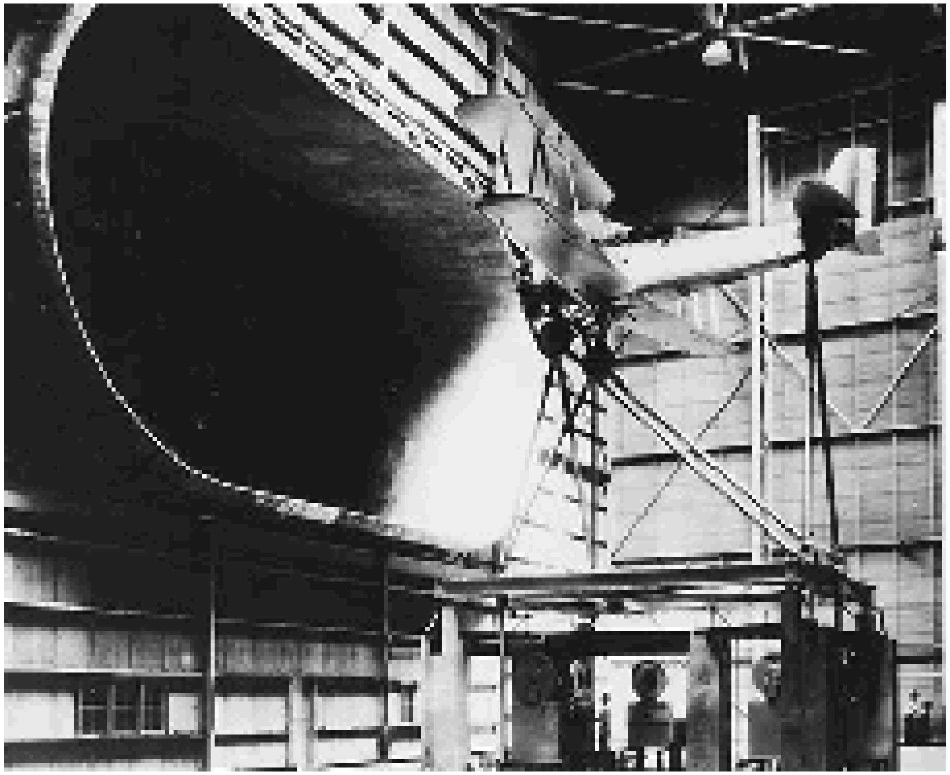
- Forces and moments are measured from a scale model at specific orientations relative to a free-stream airflow
  - Airflow can be horizontal or vertical





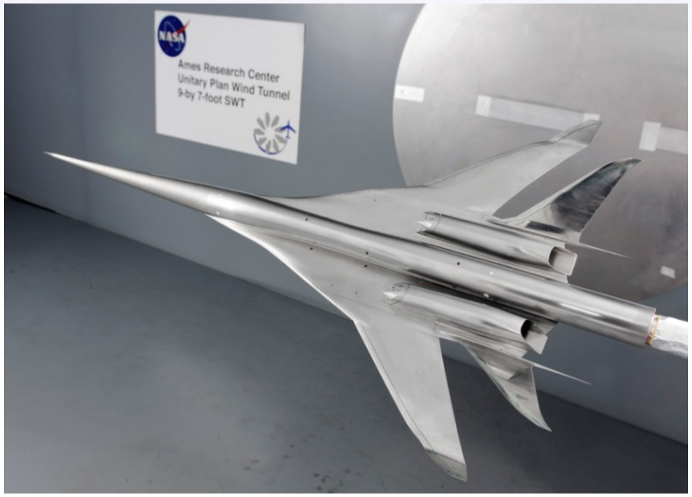
Wind-Tunnel Testing

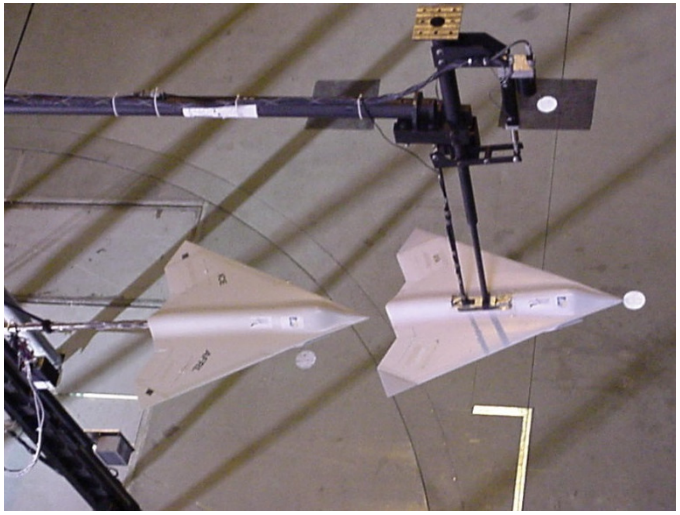
- Static Wind-Tunnel Testing
  - Model is stationary
    - Static forces, moments, hinge moments etc.



Wind-Tunnel Testing

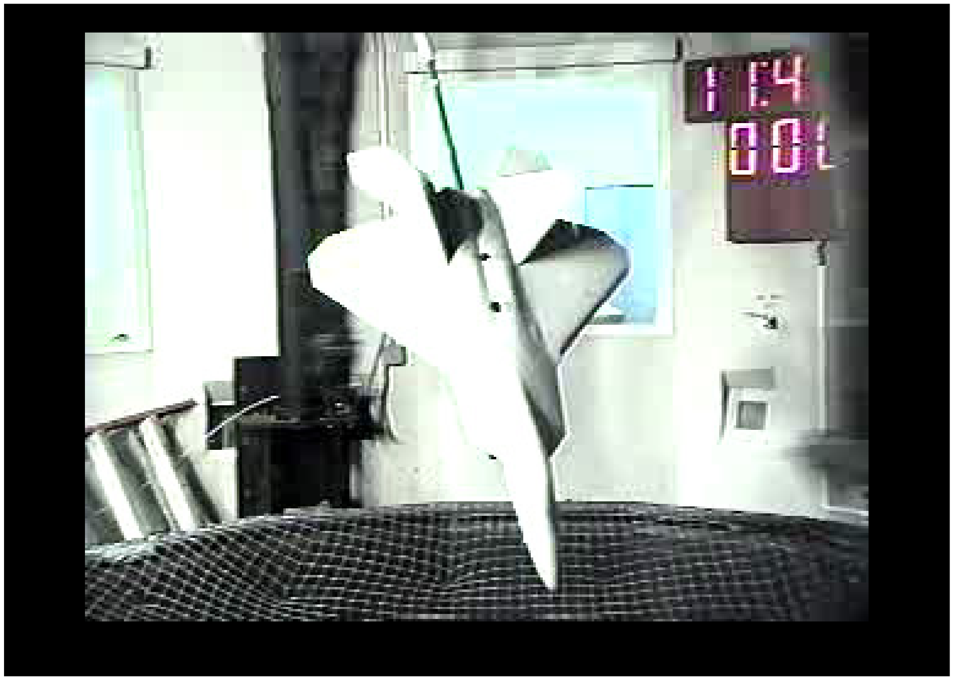
- Static Wind-Tunnel Testing
  - Model is stationary





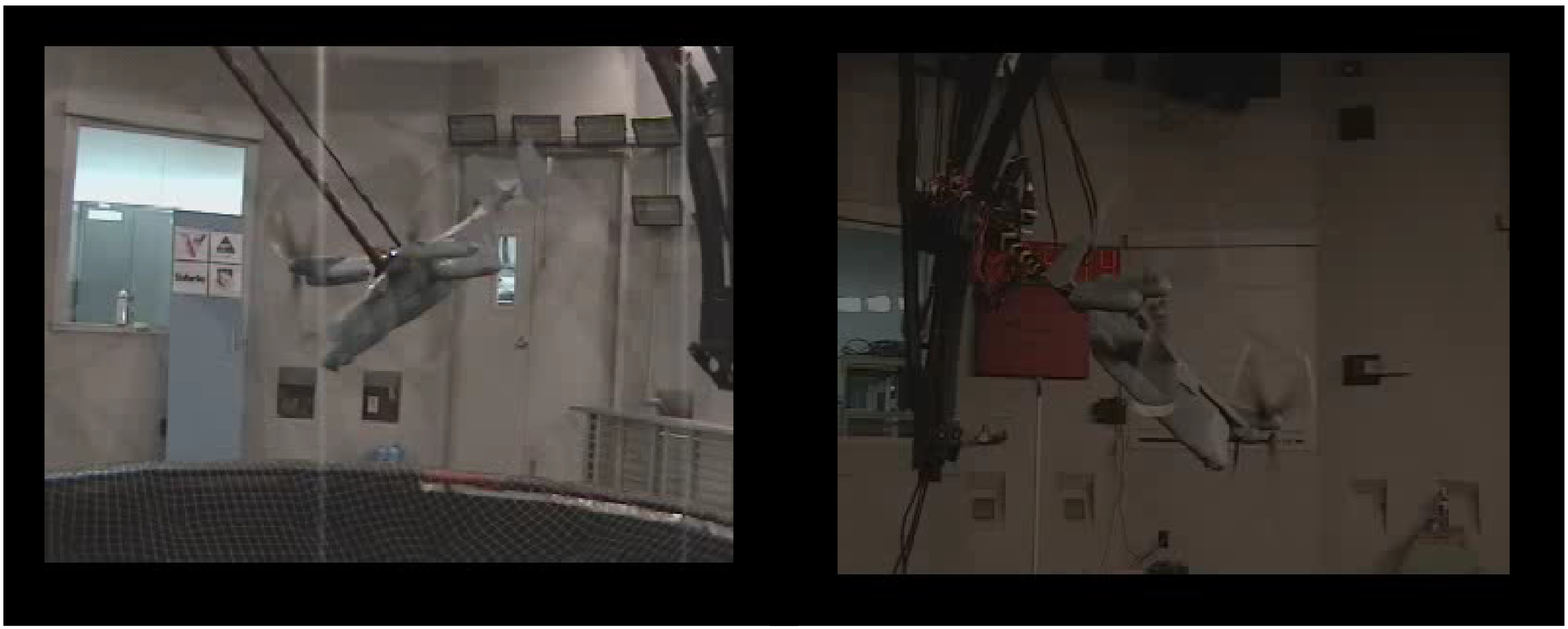
Wind-Tunnel Testing

- Dynamic Wind Tunnel Testing
  - Model is Moving
    - Dynamic forces and moments, pressure, etc.
    - Rotary Balance – Steady Rotation about velocity vector



Wind-Tunnel Testing

- Dynamic Wind Tunnel Testing
  - Model is Moving
    - Forces Oscillation – Unsteady oscillations about the body axes



Wind-Tunnel Testing

- Dynamic Wind Tunnel Testing
  - Model is Moving

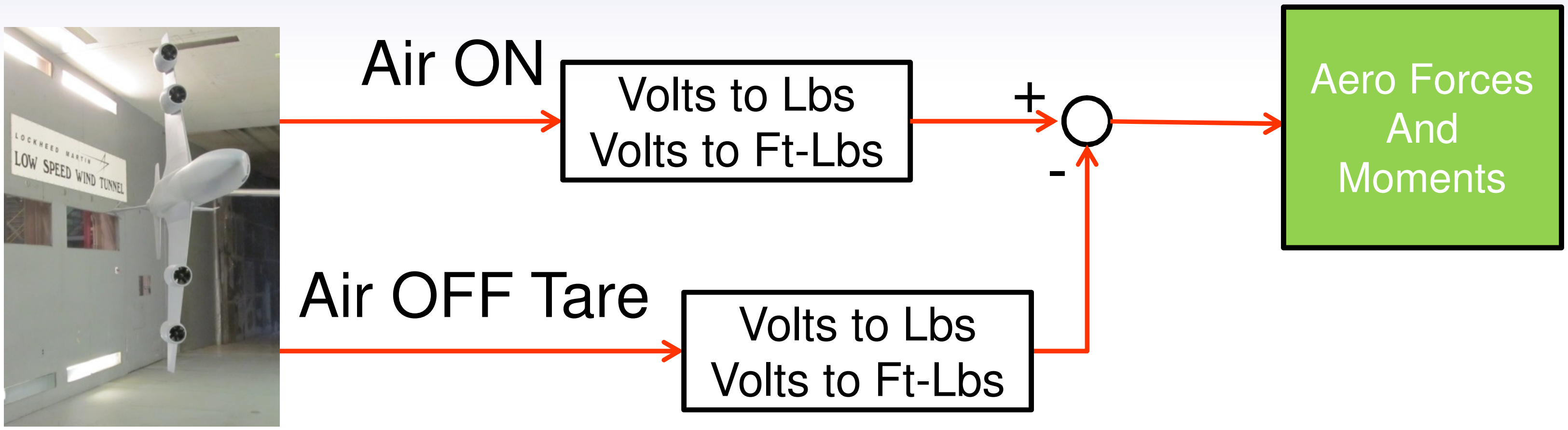
Wind-Tunnel Testing

- Test Plan
  - Definition of ALL test points
  - Critical in data reduction

SERIES	CONFIGURATION	q psf	TEST	MODE	AOA sched	LEF 1, 2	TEF 1, 2, 3, 4	SLATS 1 - 8	SPOILERS 8, 9, 10, 11	ELEVATOR L/R	STABILIZER L/R	AILERON Left Only	RUDDER
										STATIC TEST RUNS			
1	BWHV	0	Static	Tares	A0	0	0	0	0	0	0	0	0
2	BWHV	10	Static	Air-on	A0	0	0	0	0	0	0	0	0
3	BWHV	10	Static	Air-on	A1	0	0	0	0	-20	0	0	0
4	BWHV	10	Static	Air-on	A1	0	0	0	0	-15	0	0	0
5	BWHV	10	Static	Air-on	A1	0	0	0	0	-10	0	0	0
6	BWHV	10	Static	Air-on	A1	0	0	0	0	10	0	0	0
7	BWHV	10	Static	Air-on	A1	0	0	0	0	15	0	0	0
8	BWHV	10	Static	Air-on	A1	0	0	0	0	20	0	0	0
9	BWHV	10	Static	Air-on	A1	0	0	0	0	23	0	0	0
10	BWHV	10	Static	Air-on	A1	0	0	0	0	0	-4	0	0
11	BWHV	10	Static	Air-on	A1	0	0	0	0	-20	0	0	0
12	BWHV	10	Static	Air-on	A1	0	0	0	0	-15	0	0	0
13	BWHV	10	Static	Air-on	A1	0	0	0	0	-10	0	0	0
14	BWHV	10	Static	Air-on	A1	0	0	0	0	10	0	0	0
15	BWHV	10	Static	Air-on	A1	0	0	0	0	15	0	0	0
16	BWHV	10	Static	Air-on	A1	0	0	0	0	20	0	0	0
17	BWHV	10	Static	Air-on	A1	0	0	0	0	23	0	0	0
18	BWHV	10	Static	Air-on	A1	0	0	0	0	0	0	-20	0
19	BWHV	10	Static	Air-on	A1	0	0	0	0	0	0	-15	0
20	BWHV	10	Static	Air-on	A1	0	0	0	0	0	0	-10	0
21	BWHV	10	Static	Air-on	A1	0	0	0	0	0	0	-5	0
22	BWHV	10	Static	Air-on	A1	0	0	0	0	0	0	0	10
23	BWHV	10	Static	Air-on	A1	0	0	0	0	0	0	0	20
24	BWHV	10	Static	Air-on	A1	0	0	0	0	0	0	0	29

Wind-Tunnel Testing

- Data Reduction



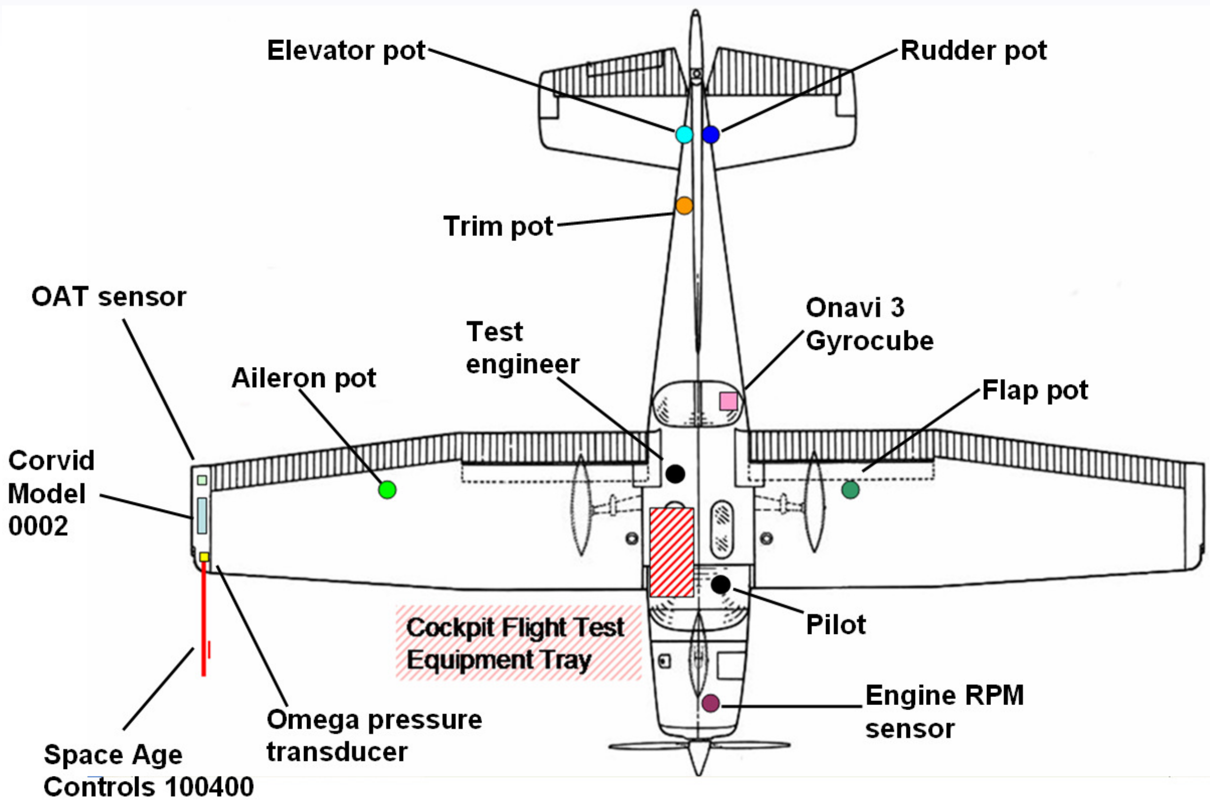
- Component increments must be computed for state, control, etc.
  - Example
    - $\Delta F_{Z_{Flap=20}} = F_{Z_{Flap=20}} - F_{Z_{Flap=0}}$
  - Resulting data are placed into tables

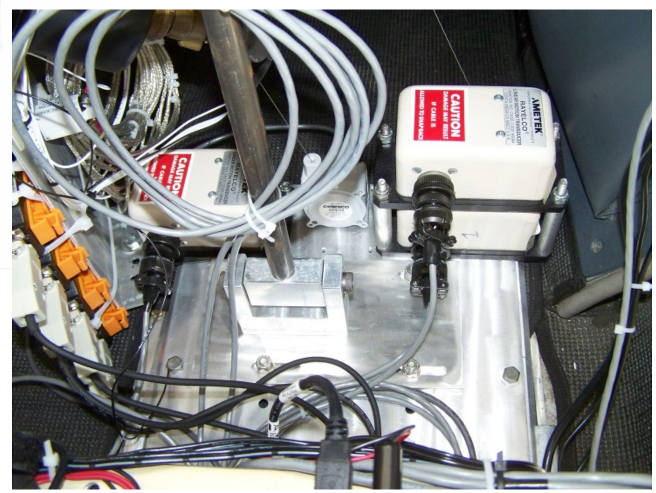
Flight-Test Data

- Data collected from a flight vehicle during the execution of specifically designed maneuvers to exploit specific vehicle behavior
  - Measure flight vehicle rates, accelerations, air speed, flow angles, control positions, engine parameters
  - Instrumentation is CRITICAL
    - Proper installation and calibration ensures good data
    - Avoid “Garbage In = Garbage Out” scenario

Flight-Test Data

– Instrumentation





Flight-Test Data

- Test Plan – Flight Modeling and Validation
  - Critical to isolate “effect” as much as possible
    - Models can only represent what can be extracted from data.
  - Trim Points
    - Flaps, Gear, Trim Settings
  - Transients
    - Flaps, Gear, Throttle
  - Dynamic
    - Isolated Control Doublets, Sine-Sweeps

Flight-Test Data

- Test Plan – Flight Modeling and Validation
  - Example: Cessna 172

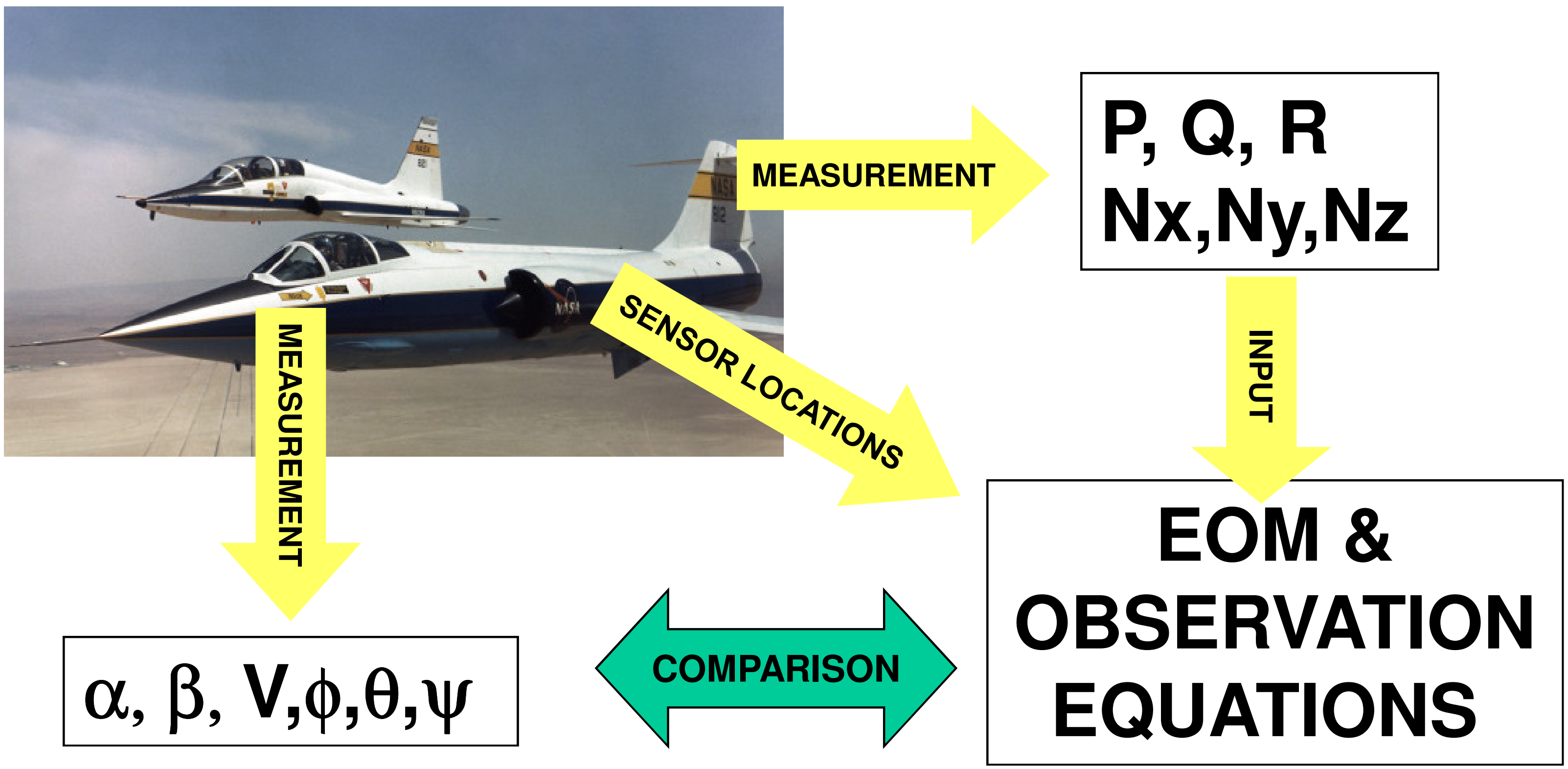
Test	Purpose
Takeoff	Ground acceleration, initial climb performance
Rejected Takeoff	Braking performance
Glide	Parasite drag
Normal Climb Performance	Climb performance
Engine Acceleration	Parameter ID and engine thrust characteristics
Engine Deceleration	Parameter ID and engine thrust characteristics
Throttle Lever Position vs. RPM	Engine performance
Power Change Force	Forces and moments created by a change in thrust
Power Change Dynamics	Dynamic changes in flight path created by change in thrust
Flap Change Force	Forces and moments created by a change in flap deflection
Flap Change Dynamics	Dynamic changes in flight path created by change in flap deflection
Longitudinal Trim	Steady-state flight conditions
Longitudinal Maneuvering Stability	Stick force/g
Longitudinal Static Stability	Level of positive static stability
Stall (Wings Level)	Stall speed and characteristics
Stall (60 deg Bank)	Stall speed and characteristics at 2g
Phugoid Dynamics	Dynamic longitudinal stability
Short Period Dynamics	Dynamic longitudinal and lateral-directional stability and parameter ID
Roll Response (Rate)	Roll rate of aircraft
Roll Response to Step Input	Roll overshoot tendencies
Spiral Stability	Lateral roll stability
Rudder Response	Roll induced by rudder
Dutch Roll	Lateral directional coupling
Steady State Sideslip	Static directional stability
Windup Turns	Longitudinal maneuvering stability
Wings-Level Pull-Up	Longitudinal maneuvering stability
Spins	Dynamic behavior of aircraft during spin

Flight-Test Data

- Data Extraction
  - Focus here is on Aero-Data Extraction
    - Other data collected/extracted from flight
      - Required for Level D simulators
        - » Control System Behavior – Forces, Dynamics, etc.
        - » Engine Behavior
        - » Ground Handling
        - » Instruments behavior
        - » Vibrations and Buffet
        - » Sound
  - Aerodynamics data are extracted from measured accelerations and angular rate

Flight-Test Data

- Data Extraction
  - Data quality is critical – Data consistency and compatibility



Flight-Test Data

- Data Extraction
  - Solve for forces and moments then coefficients
    - From:

$$\dot{\boldsymbol{v}}_{\boldsymbol{B}} = \frac{1}{m}\boldsymbol{F}_{\boldsymbol{B}}\,(\boldsymbol{\omega}_{\boldsymbol{B}} \times \boldsymbol{v}_{\boldsymbol{B}}) + \boldsymbol{B}\boldsymbol{g}_{\boldsymbol{o}}$$

$$\dot{\boldsymbol{\omega}}_{\boldsymbol{B}} = -\boldsymbol{J}^{-1}\big(\boldsymbol{\omega}_B \times (\boldsymbol{J}\boldsymbol{\omega}_B)\big) + \boldsymbol{J}^{-1}\boldsymbol{T}_{\boldsymbol{B}}$$

- To:

Density

Coefficient

Ref Area

Accelerations

Attitude

Mass

Gravity

$$\boldsymbol{C}_{\boldsymbol{B}}\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\rho}\boldsymbol{V}^{\mathbf{2}}\boldsymbol{S} = \boldsymbol{F}_{\boldsymbol{B}} = m\,\frac{\dot{\boldsymbol{v}}_{\boldsymbol{B}} - \boldsymbol{B}\boldsymbol{g}_{\boldsymbol{o}}}{(\boldsymbol{\omega}_{\boldsymbol{B}} \times \boldsymbol{v}_{\boldsymbol{B}})}$$

Characteristic Length

Angular Rates

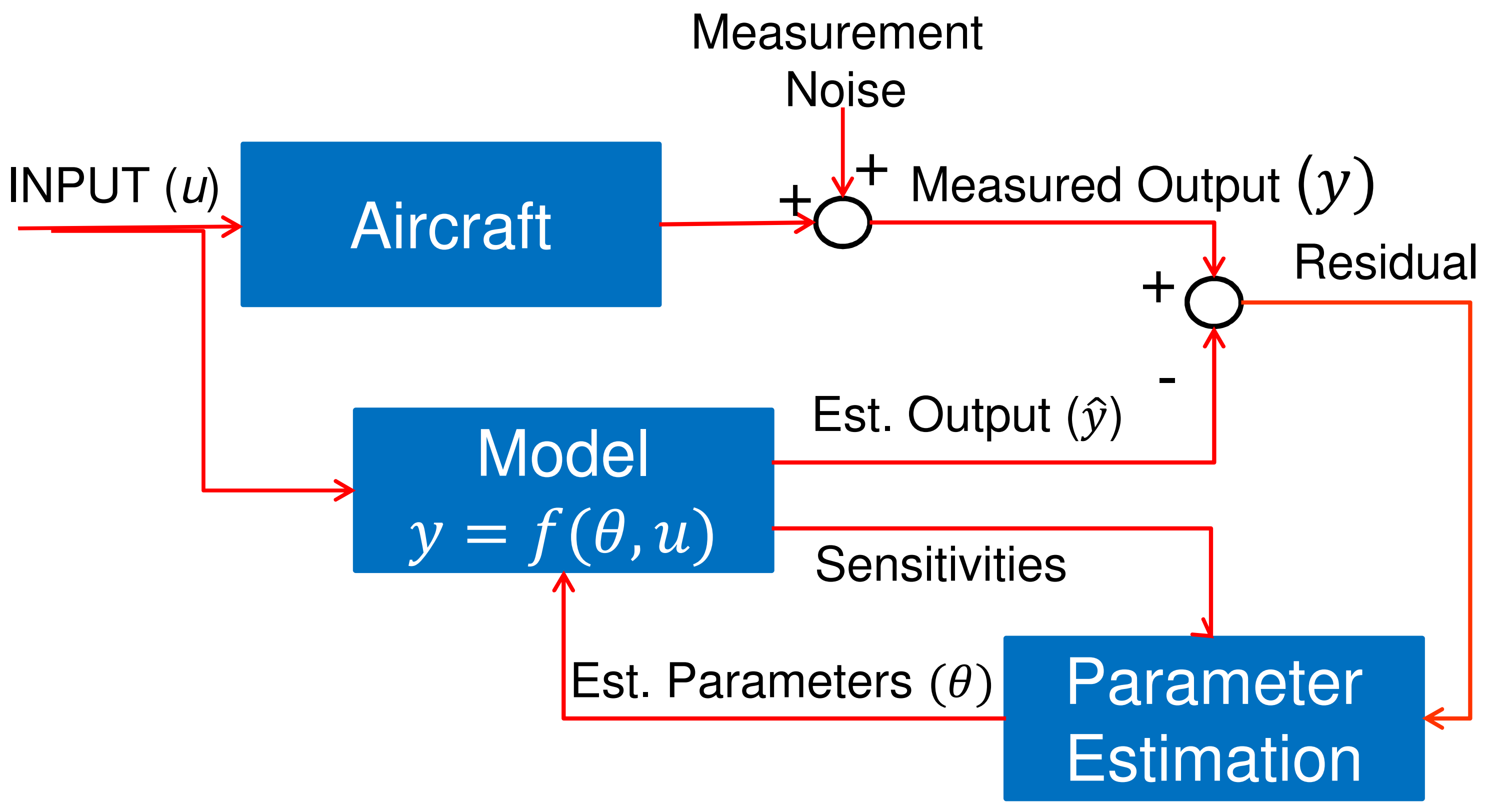
Air Speed

$$\boldsymbol{C}_{\boldsymbol{B}}\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\rho}\boldsymbol{V}^{\mathbf{2}}\boldsymbol{S}\boldsymbol{l} = \boldsymbol{T}_{\boldsymbol{B}} = \frac{\mathbf{1}}{\boldsymbol{J}^{-\mathbf{1}}}\,\big(\dot{\boldsymbol{\omega}}_{\boldsymbol{B}} + \boldsymbol{J}^{-1}\big(\boldsymbol{\omega}_B \times (\boldsymbol{J}\boldsymbol{\omega}_B)\big)\big)$$

Inertias

Flight-Test Data

- Model Identification
  - Output Error
    - Time or Frequency Domain



Flight-Test Data

- Model Identification
  - Output Error
    - Example Model

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -\frac{\bar{q}S}{mV_0}C_{L_\alpha} + \frac{g\sin(\gamma_0)}{V_0} - \frac{T_0\cos(\alpha_0)}{mV_0} & 1 - \frac{\bar{q}S\bar{c}}{2mV_0^2}C_{L_q} \\ \frac{\bar{q}S\bar{c}}{I_{yy}}C_{m_\alpha} & \frac{\bar{q}S\bar{c}^2}{2V_0I_{yy}}C_{m_q} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -\frac{\bar{c}S}{mV_0}C_{L_{\delta_e}} & \theta_7 \\ \frac{\bar{q}S\bar{c}}{I_{yy}}C_{m_{\delta_e}} & \theta_8 \end{bmatrix} \begin{bmatrix} \delta_e \\ 1 \end{bmatrix}$$

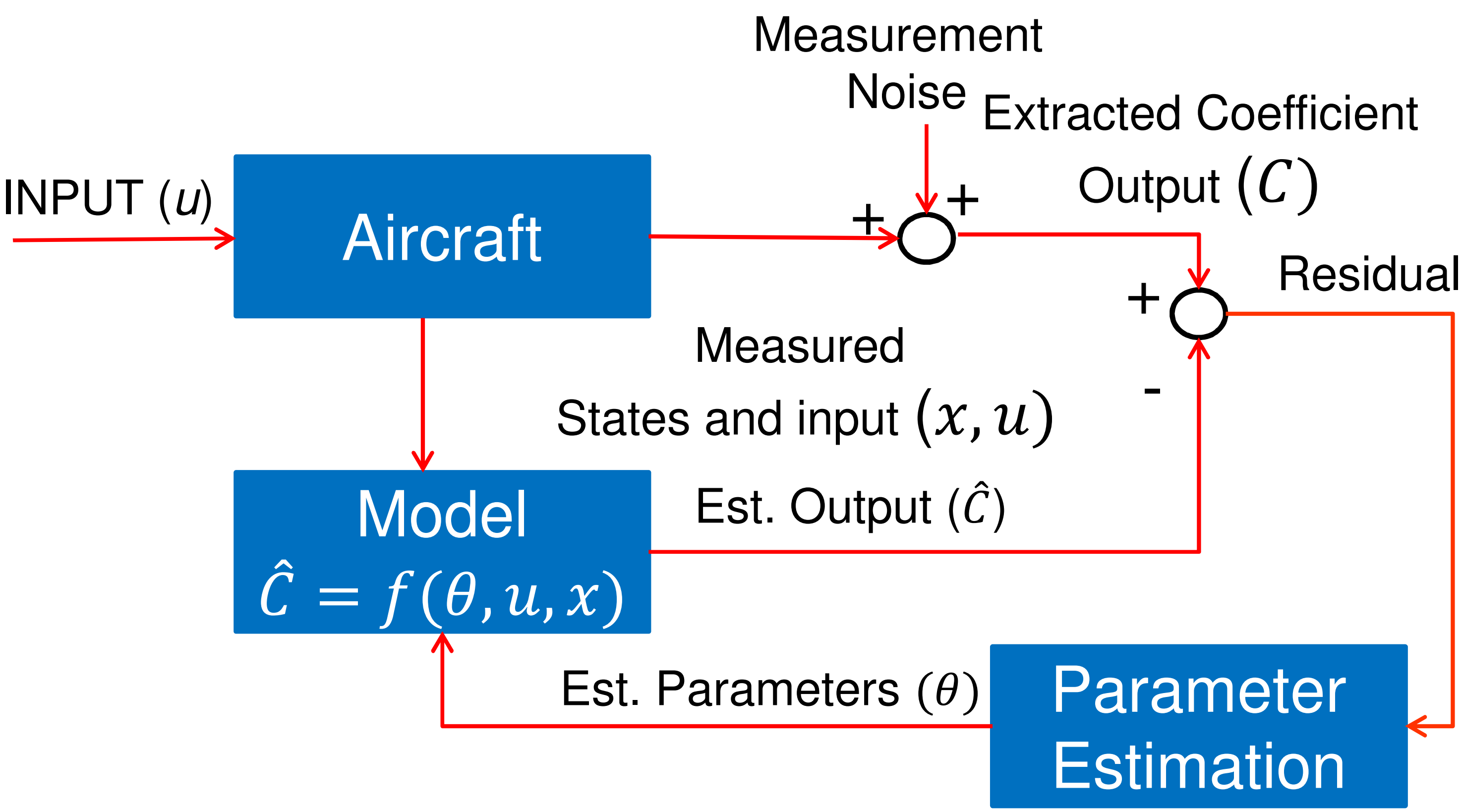
$$\begin{bmatrix} \alpha \\ q \\ a_z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{\bar{q}S}{mg}C_{L_\alpha} & -\frac{\bar{q}S\bar{c}}{2V_0mg}C_{m_q} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{\bar{q}S}{mg}C_{L_{\delta_e}} & \theta_9 \end{bmatrix} \begin{bmatrix} \delta_e \\ 1 \end{bmatrix} \qquad (21)$$

$$\boldsymbol{\theta} = \begin{bmatrix} C_{L_\alpha} & C_{L_q} & C_{L_{\delta_e}} & C_{m_\alpha} & C_{m_q} & C_{m_{\delta_e}} & \theta_7 & \theta_8 & \theta_9 \end{bmatrix}^T$$

Grouer, J.A. “Real-Time Parameter Estimation Using Output Error,” 2014, NASA Report Number NF1676L-17743

Flight-Test Data

- Model Identification
  - Equation Error
    - Time Domain



Flight-Test Data

- Model Identification
  - Equation Error
    - Sample Model

$$C_{m_{Aero}} = (C_{m_o} + C_{m_\alpha}\alpha + C_{m_{de}}de + C_{m_q}\frac{q\bar{c}}{2V})$$

$$\theta = \left[C_{m_o}, C_{m_\alpha}, C_{m_{de}}, C_{m_q}\right]$$

Summary

- Sources for data vary in capability, complexity and cost
  - Computation Methods
    - Predictive methods exist for a good start to simulation development
    - CFD may be too cumbersome for most flight simulation development activities
  - Wind-Tunnel Testing
    - Tests can provide comprehensive data base
    - Advanced techniques exist for modeling
  - Flight Data
    - Truth data
    - Instrumentation and data quality are critical for model extraction from flight.
    - Model is limited to extents of the testing

QUESTIONS?