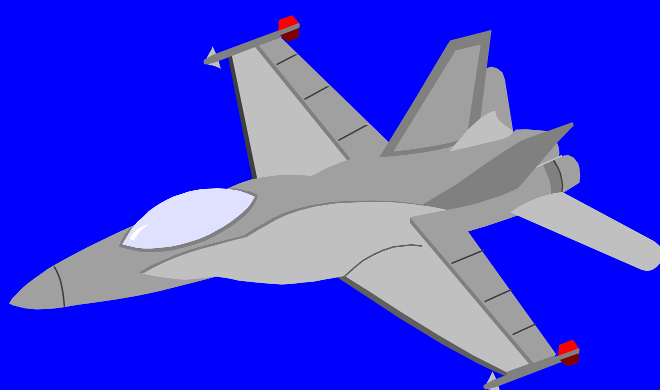
MATHEMATICAL MODELING
OF
VEHICLE DYNAMICS



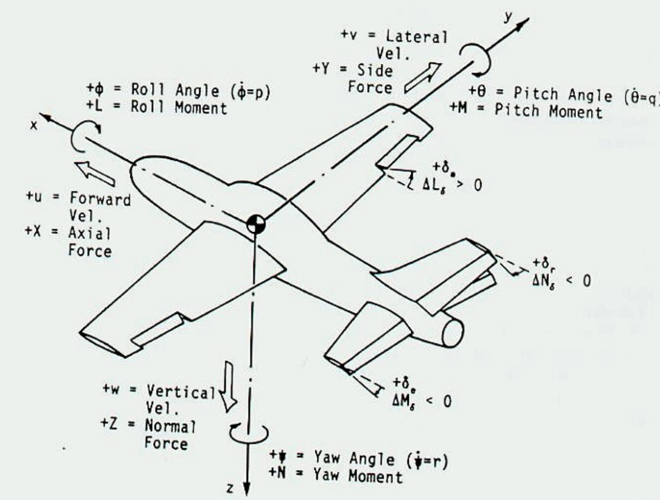


Fig. 1.1 Coordinate sign convention.

VEHICLE DYNAMICS
MATH MODELING & VALIDATION QUESTIONS

- DOES IT FLY LIKE THE AIRPLANE?
  - How can you tell?
  - How much do you care?
- WHAT IS “DATA”?
- WHERE CAN I GO WRONG?
- HOW CLOSE IS CLOSE ENOUGH?
- ‘BAND-AIDS’ OR MAJOR SURGERY?
- WHO IS RIGHT… THE PILOT OR THE DATA?

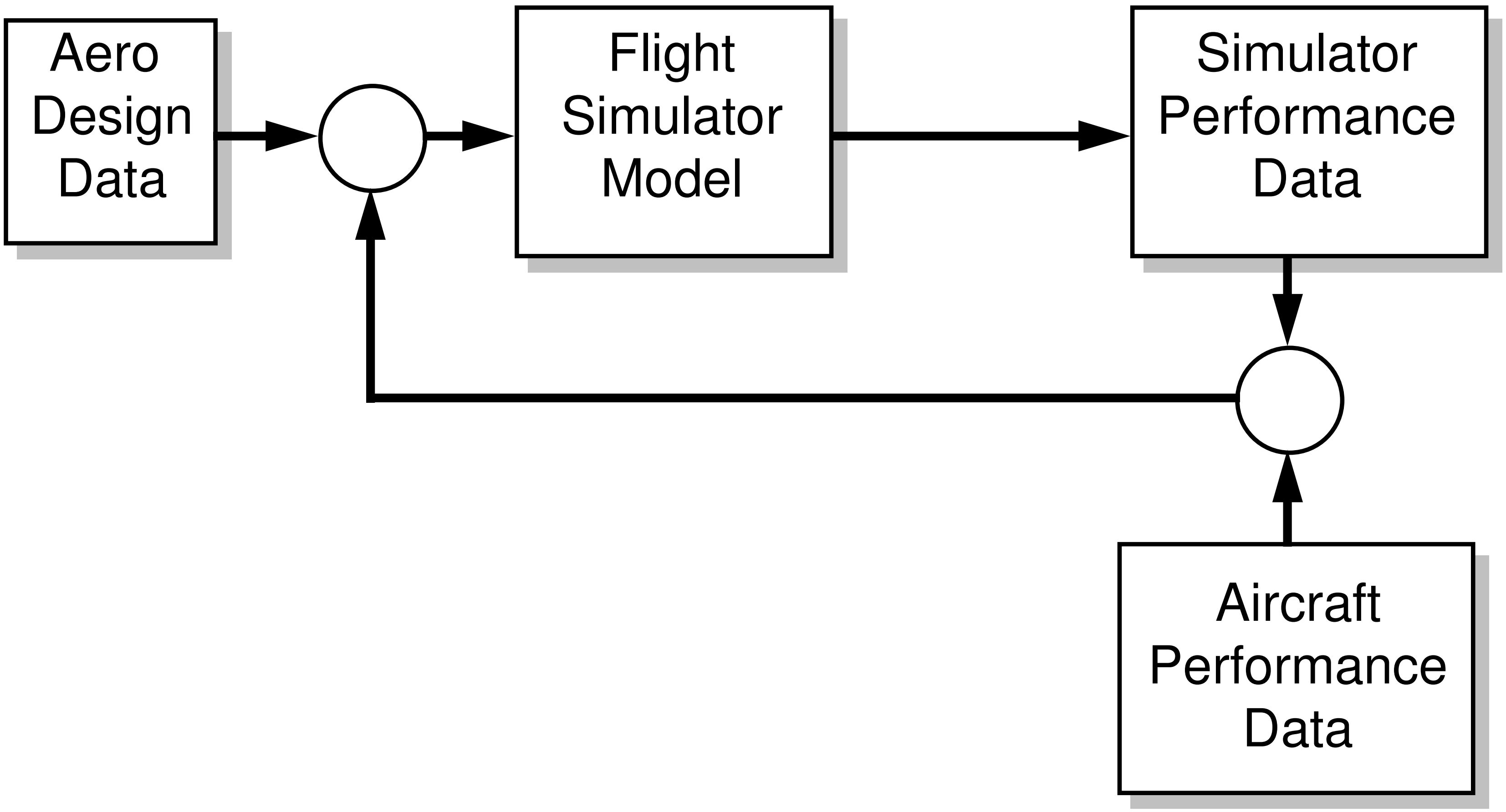
MATH MODELING

PURPOSE: Overview of Common Practices

TOPICS:

- Modeling Fundamentals
- Aerodynamic Math Models
- Data for Models
- Other Models
- Flight Test Validation Data

FLIGHT SIMULATION
Modeling & Validation



M&S TERMINOLOGY

per DODD 5000.59

- MODEL: mathematical representation of a system
- SIMULATION: software framework for executing models *(more than software!!)*
- MODELING & SIMULATION: An analytical problem solving approach
- VERIFICATION: M&S *capability*
- VALIDATION: M&S *credibility*

Ref: DMSO VV&A Recommended Practices Guide, August 2001

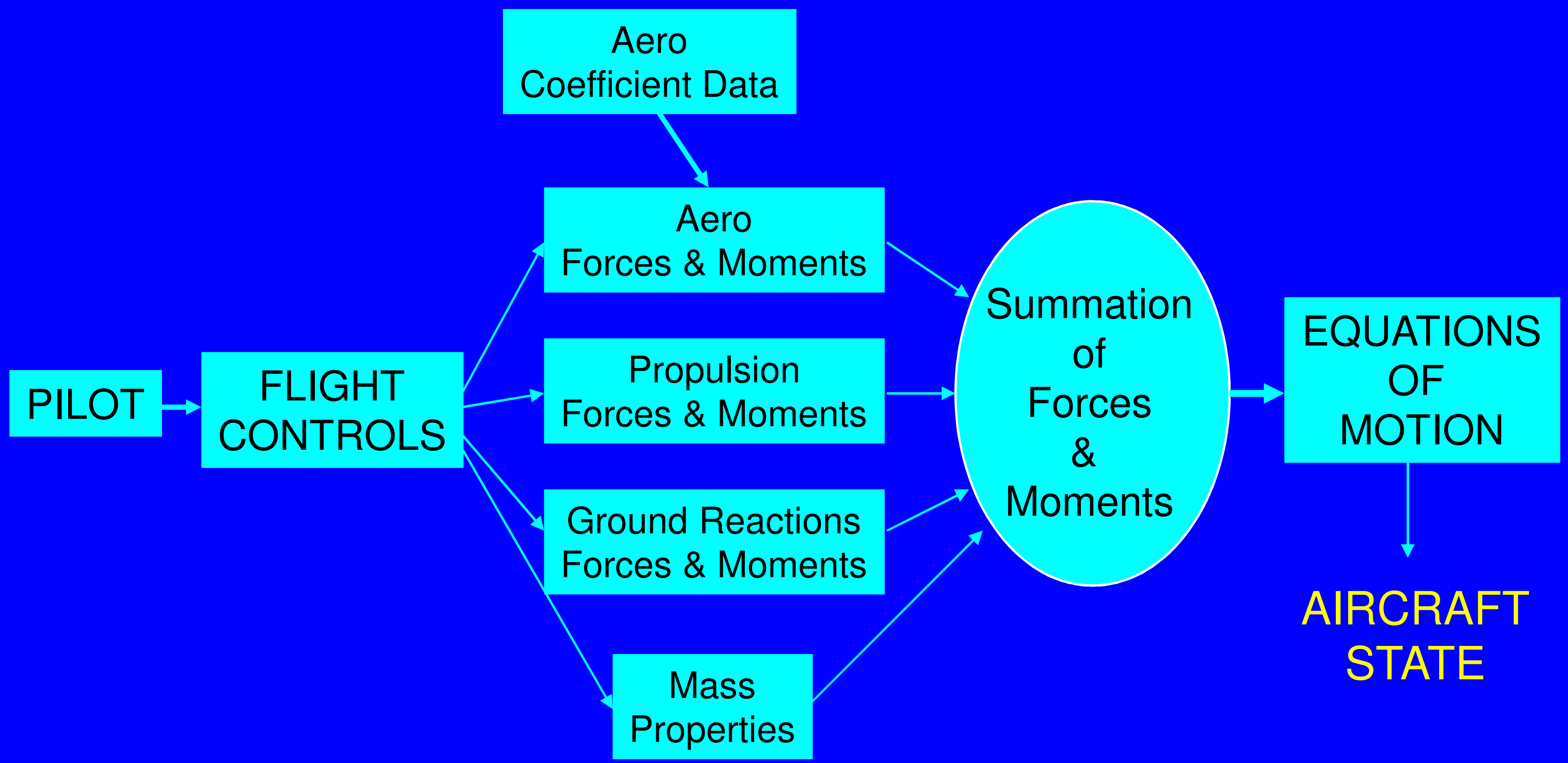
SYSTEM CHARACTERISTICS
Modeling Assumptions

- <u>Deterministic</u> - Stochastic
- <u>Continuous</u> - Discrete*
- Linear* - <u>Nonlinear</u>
- <u>Stationary</u> - Nonstationary
- Degrees of Freedom (6, 5, 3, 6+)
- Rigid Body
- Constant Mass

MATH MODELING METHODOLOGY

- Apply basic physical laws (Newton...)
- Write equations in engineering terms
- Document assumptions, simplifications
- Solve for the dependent variables
- Code the resulting equations

TYPICAL FLIGHT SIMULATION MODEL



TYPES OF DYNAMICS MODELS

- Force & Moment Models (physics)
- Perturbation Models (linear analysis)
- Kinematic Models (simple, no F&M)

FORCE & MOMENT EOM
6 DOF Model

- Sum Forces (3 translational axes)
- Sum Moments (3 rotational axes)
- Solve for Acceleration in each Axis
- Integrate for Velocity
- Integrate again for Position, Attitude
  - Vehicle State

FORCE & MOMENT MODEL
Basic Physics Solution Process

$$F = m\,a$$

a: acceleration
v: velocity
x: position

$$a = F / m$$

then:

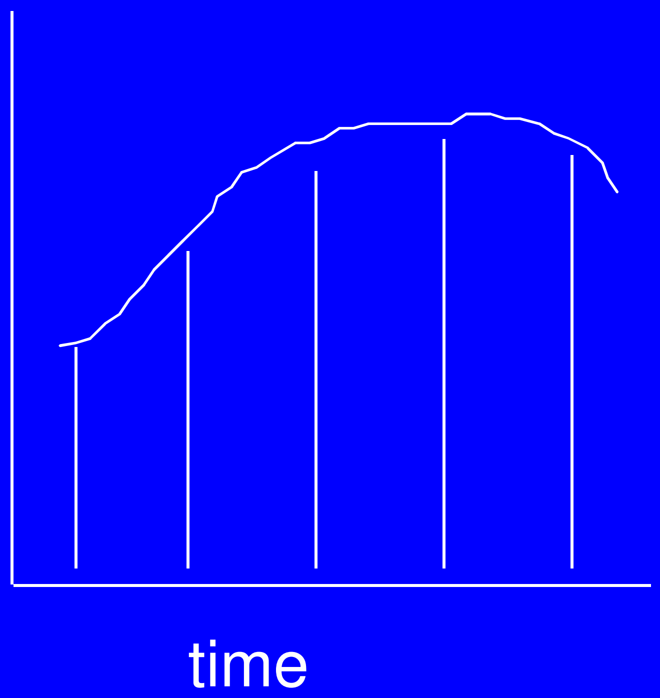
$v = \int a\,dt$

$x = \int v\,dt$

NUMERICAL INTEGRATION

Acceleration → Velocity → Position

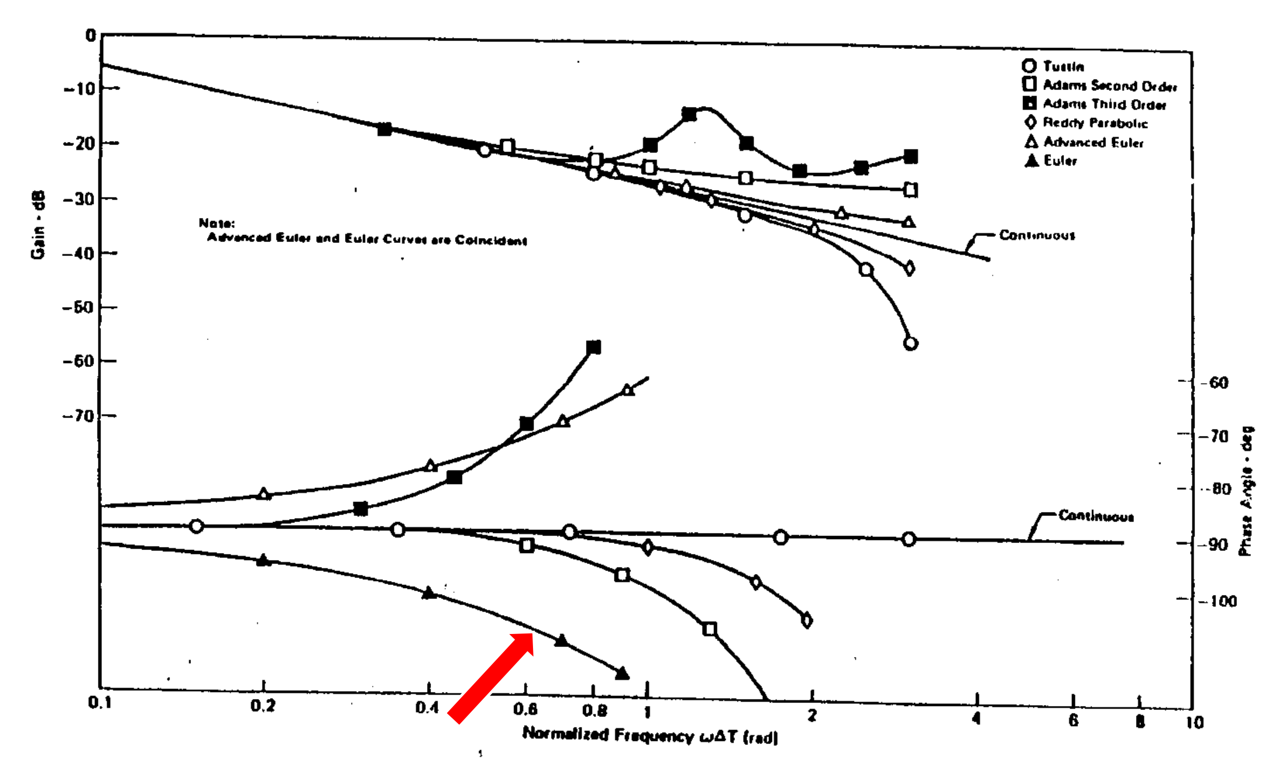
- Many Algorithms
- Algorithm Factors
  - Stability
  - Accuracy
  - Speed
  - Time distortion
- Typical: simple Euler
  - $X_n = X_{n-1} + xdot_{n-1} * dt$



DYNAMIC INFLUENCE OF INTEGRATION ALGORITHMS

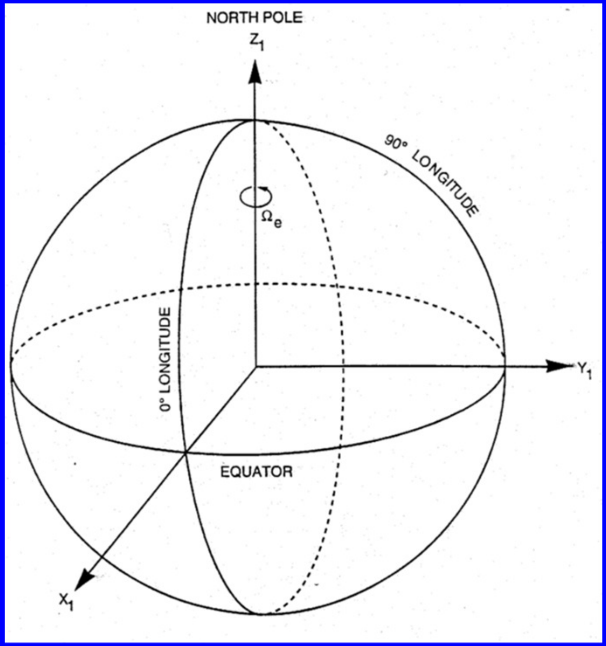
Euler (Rectangular)	$X_N = X_{N-1} + \dot{X}_{N-1}\,\Delta T$	Zero Order Predictor
Advanced Euler (Z-Transform)	$X_N = X_{N-1} + \dot{X}_N\,\Delta T$	Zero Order Corrector
Adams Second Order (Trapezoidal)	$X_N = X_{N-1} + (1.5\dot{X}_{N-1} - 0.5\dot{X}_{N-2})\,\Delta T$	Linear Predictor
Tustin	$X_N = X_{N-1} + (0.5\dot{X}_N + 0.5\dot{X}_{N-1})\,\Delta T$	Linear Corrector
Adams Third Order	$X = X_{N-1} + (\frac{23}{12}\dot{X}_{N-1} - \frac{4}{3}\dot{X}_{N-2} + \frac{5}{12}\dot{X}_{N-3})\,\Delta T$	Parabolic Predictor
Reddy Parabolic	$X = X_{N-1} + (\frac{5}{12}\dot{X}_N + \frac{2}{3}\dot{X}_{N-1} - \frac{1}{12}\dot{X}_{N-2})\,\Delta T$	Parabolic Corrector

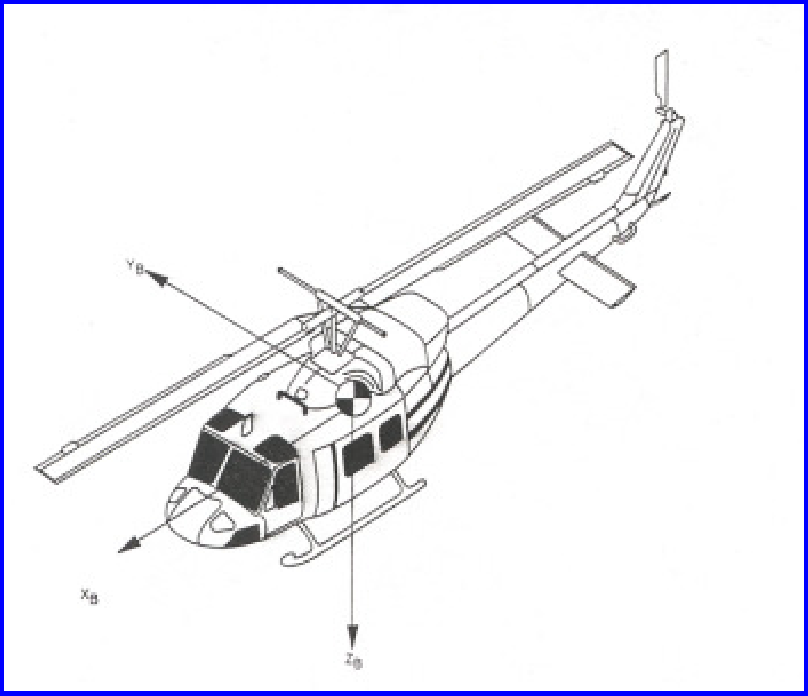
NOTE PHASE LAG INDUCED BY EULER ALGORITHM IF △T LARGE (low iteration rate)



For 20 Hz iteration rate:
A pilot input of 2 Hz incurs
17 degrees more phase lag
than in real world physics

COORDINATE SYSTEMS
&
COORDINATE TRANSFORATIONS





Where am I?
What's my attitude?

COORDINATE TRANSFORMATIONS

- Purpose: Relate fixed & rotating axis systems
- Method: Direction Cosine Matrix
  - Euler Angle Method
  - Quaternion Method
- Result: Vehicle Orientation
  - Euler Angles (Yaw, Pitch, Roll)

EQUATIONS OF MOTION
BASIC COORDINATE TRANSFORMATION

DIRECTION COSINE MATRIX

Fixed - Moving Axes Transformation

$$[M] = \begin{matrix} a1 & a2 & a3 \\ b1 & b2 & b3 \\ c1 & c2 & c3 \end{matrix}$$

$$a1 = \cos\Theta \cos\Psi$$

$$a2 = \cos\Theta \sin\Psi$$

$$a3 = -\sin\Theta$$

$$b1 = \sin\Phi \sin\Theta \cos\Psi - \cos\Phi \sin\Psi$$

$$b2 = \sin\Phi \sin\Theta \sin\Psi + \cos\Phi \cos\Psi$$

$$b3 = \sin\Phi \cos\Theta$$

$$c1 = \cos\Phi \sin\Theta \cos\Psi + \sin\Phi \sin\Psi$$

$$c2 = \cos\Phi \sin\Theta \sin\Psi - \sin\Phi \cos\Psi$$

$$c3 = \cos\Phi \cos\Theta$$

Fixed axis: Earth

Moving axis: Aircraft

Euler Angle Method

- Solve the Rate Equations

Yaw: $\Psi dot = (r\cos\Phi + q\sin\Phi)\sec\Theta$

Pitch: $\Theta dot = q\cos\Phi - r\sin\Phi$

Roll: $\Phi dot = p + q\tan\Theta\sin\Phi + r\tan\Theta\cos\Phi$

- Integrate the Rate Equations
- Populate the Direction Cosine Matrix

NOTE: At Pitch= +/- 90 deg, Roll & Yaw Rate undefined!
Must implement work around in EOM software.

QUATERNION METHOD

Ref: ANSI/AIAA R-004-1992

Quaternions defined from Euler angles

$$e_0 = \cos\psi/2\ \cos\theta/2\ \cos\phi/2 + \sin\psi/2\ \sin\theta/2\ \sin\phi/2$$

$$e_1 = \cos\psi/2\ \cos\theta/2\ \sin\phi/2 - \sin\psi/2\ \sin\theta/2\ \cos\phi/2$$

$$e_2 = \cos\psi/2\ \sin\theta/2\ \cos\phi/2 + \sin\psi/2\ \cos\theta/2\ \sin\phi/2$$

$$e_3 = -\cos\psi/2\ \sin\theta/2\ \sin\phi/2 + \sin\psi/2\ \cos\theta/2\ \cos\phi/2$$

Quaternion derivatives defined from vehicle angular velocities

$$\dot{e}_0 = -\frac{1}{2}(e_1 p + e_2 q + e_3 r) + K\varepsilon e_0$$

$$\dot{e}_1 = \frac{1}{2}(e_0 p + e_2 r - e_3 q) + K\varepsilon e_1$$

$$\dot{e}_2 = \frac{1}{2}(e_0 q + e_3 p - e_1 r) + K\varepsilon e_2$$

$$\dot{e}_3 = \frac{1}{2}(e_0 r + e_1 q - e_2 p) + K\varepsilon e_3$$

Transformation matrix definitions

$$a_1 = (e_0^2 + e_1^2 - e_2^2 - e_3^2)$$

$$a_2 = 2(e_1 e_2 + e_0 e_3)$$

$$a_3 = 2(e_1 e_3 - e_0 e_2)$$

$$b_1 = 2(e_1 e_2 - e_0 e_3)$$

$$b_2 = (e_0^2 + e_2^2 - e_1^2 - e_3^2)$$

$$b_3 = 2(e_2 e_3 + e_0 e_1)$$

$$c_1 = 2(e_1 e_3 + e_0 e_2)$$

$$c_2 = 2(e_2 e_3 - e_0 e_1)$$

$$c_3 = (e_0^2 + e_3^2 - e_1^2 - e_2^2)$$

Warning for model reuse! Definition details vary.

MULTIPLE COORDINATE SYSTEMS APPLIED

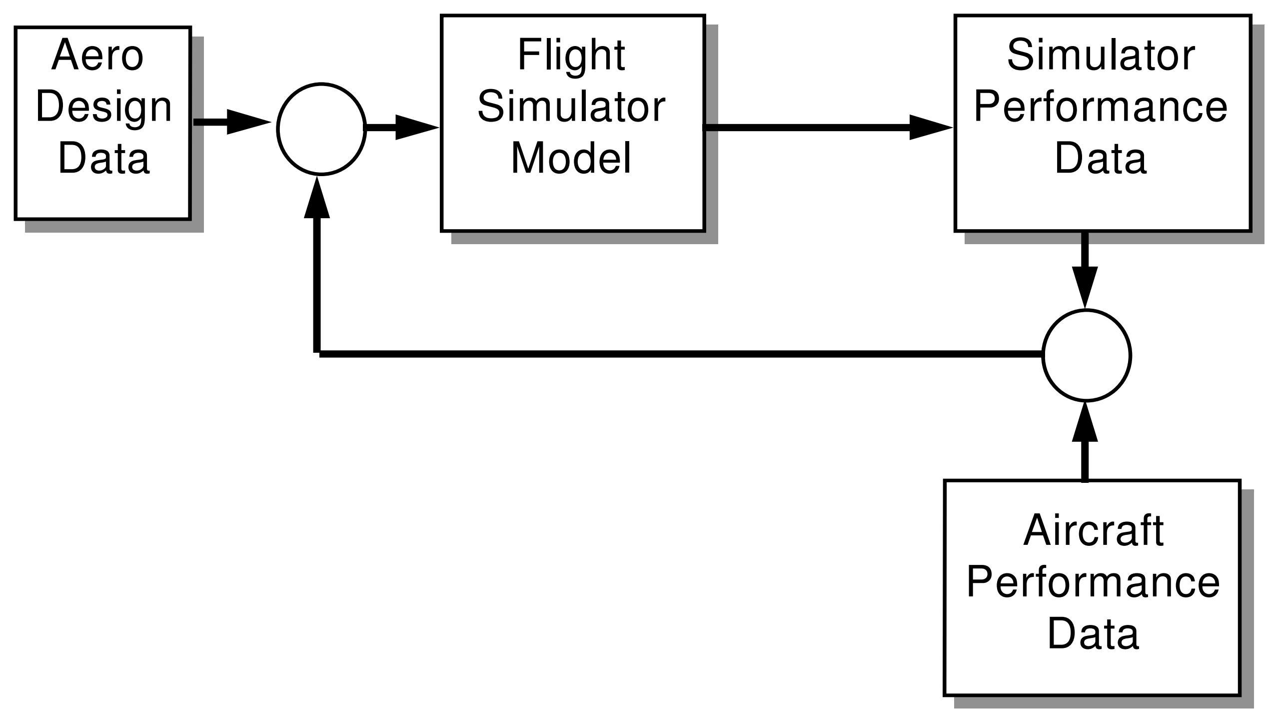
- Inertial
- Earth / NED
- Body *(attached to each vehicle)*
- Flat Earth *(typical visual database)*
- WGS84 *(DMA, NIMA system)*
- and many more…..

MATH MODELING RISK AREAS
(even before the Aero part!)

- Assumptions, simplifications
- Omissions, sign errors
  - Coordinate Systems, Transformations
  - Equations of Motion (EOM)
  - Numerical Integration

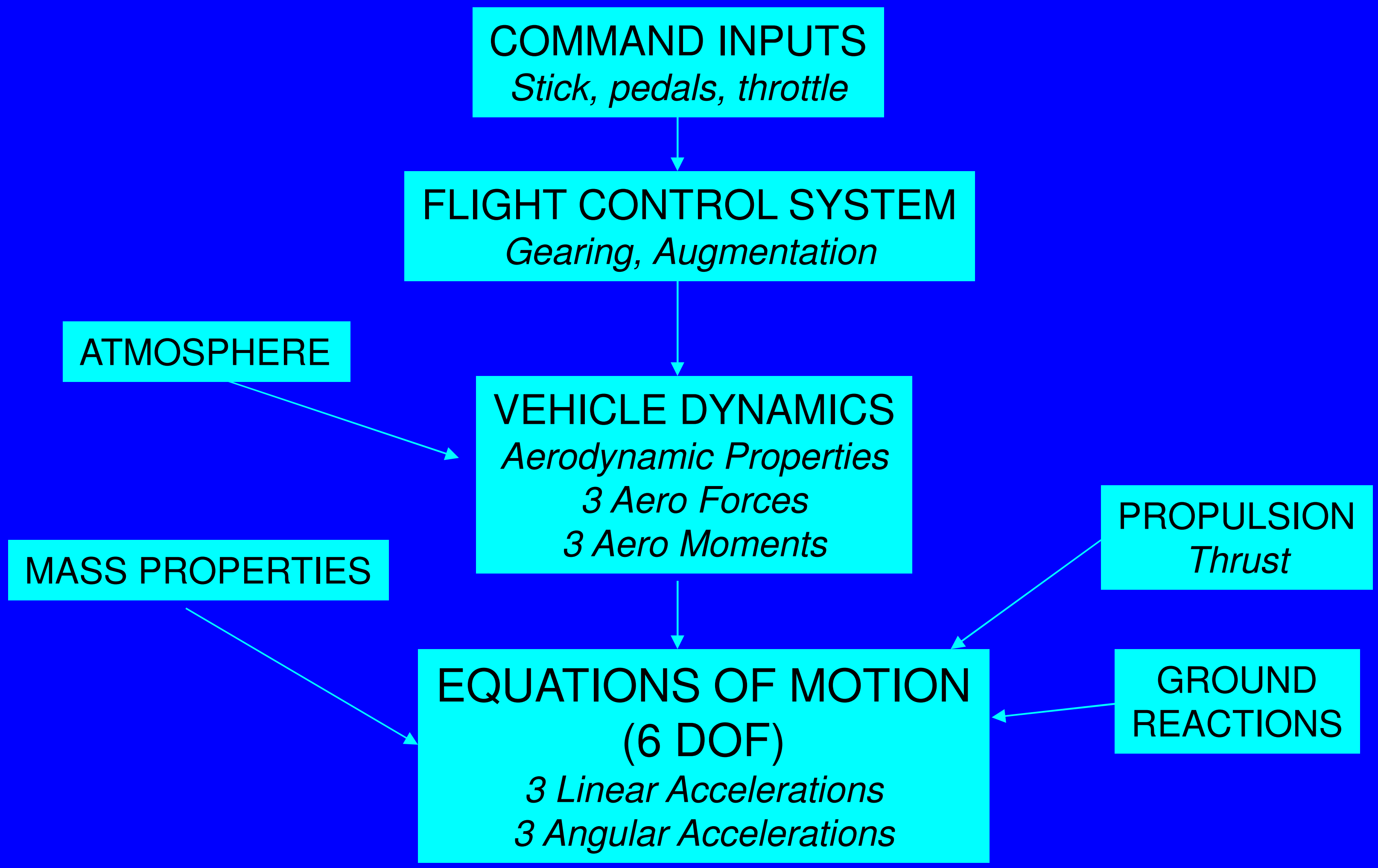
AERODYNAMIC MATH MODELING
FOR
FLIGHT SIMULATORS

MODELING COMPONENTS

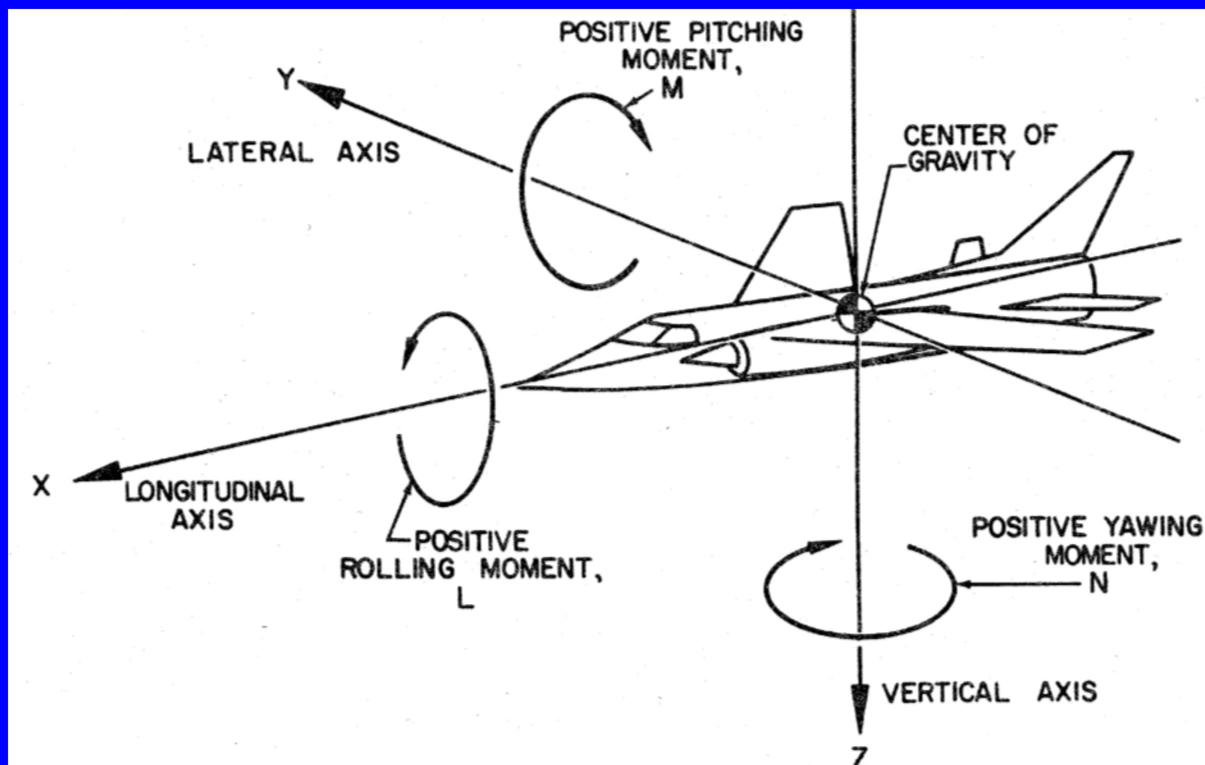


NOTE: 1 “Model” + 3 “Data”

6 DOF FORCE & MOMENT MODEL



AIRPLANE BODY AXIS SYSTEM



Typical Aero Sign Convention

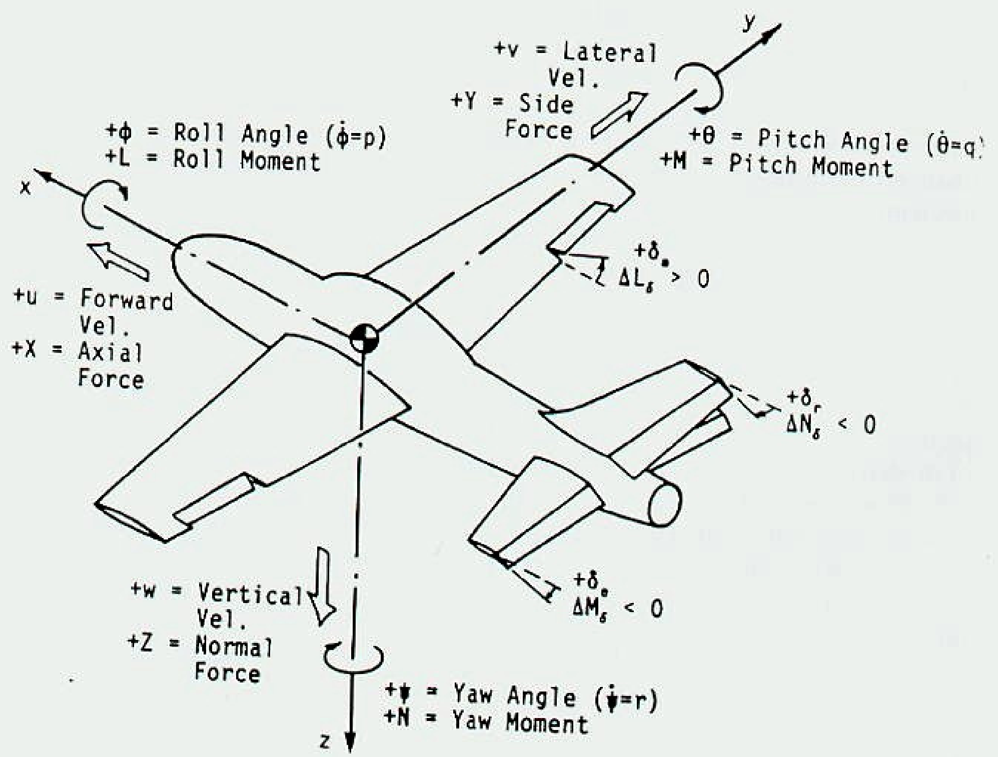


Fig. 1.1 Coordinate sign convention.

Math model documentation must provide definitions!

6 DOF Equations of Motion

Complete expressions – valid for large, non-linear excursions

Force Equations

X – mg sinθ = m (**udot** + qv –rv)
Y + mg cosθ sinϕ = m (**vdot** + ru –pw)
Z + mg cosθ cosϕ = m (**wdot** + pv –qu)

Moment Equations

L = Ix **pdot** – Ixz rdot + qr (Iz – Iy) – Ixz pq
M = Iy **qdot** + rp (Ix – Iz) + Ixz (p^2 – r^2)
N = Iz **rdot** – Ixz pdot + pq (Iy – Ix) + Ixz qr

Where:
u, v, w are body translational velocities
p, q, r are body angular velocities
Ixy, Iyz are normally zero due to body symmetry
mg terms are gravity force components
θ, ϕ are pitch and roll Euler angles

AERO FORCE & MOMENT COEFFICIENTS

BODY AXIS

FORCES

$FX = C_X * Q\,S$

$FY = C_Y * Q\,S$

$FZ = C_Z * Q\,S$

MOMENTS

$MX = C_l * QSb$

$MY = C_m * QSc$

$MZ = C_n * QSb$

Coefficients: Dimensionless

Q (dynamic pressure: lb/sq ft

S (wing area): sq ft

b (wing span: ft

c (mean aerodynamic chord) - ft

AERODYNAMIC DATA SOURCES

- Wind Tunnel
  - Static Tests
    - Lift, drag, moments
  - Dynamic Tests
    - Rotary (spin characteristics)
    - Forced oscillation (dynamic damping)
- Analytical Methods
  - Computational Fluid Dynamics
  - Empirical
    - Public domain software (USAF DATCOM*)
    - Commercial software (e.g., AAA** by DARCORP)
- Flight Test Data Extraction
  - Parameter/System Identification (PID) software

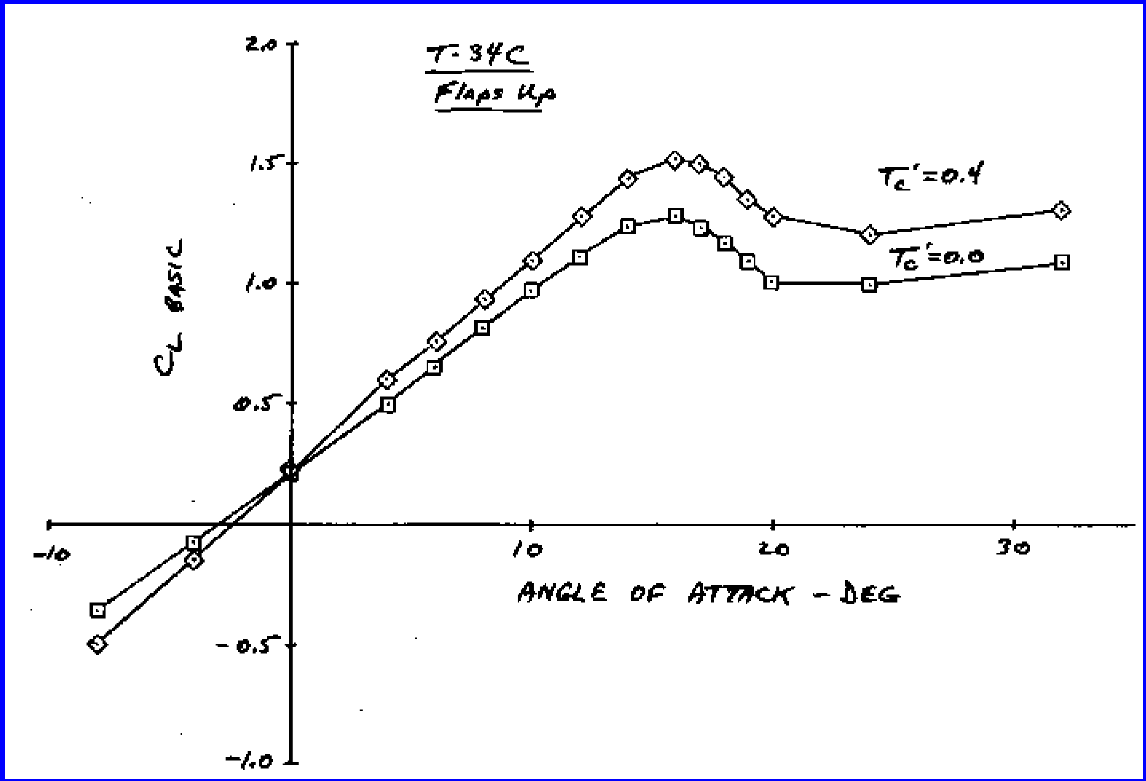
*DATCOM: DATa COMpendium, AFFDL-TR-79-3032
**AAA: Advanced Aircraft Analysis

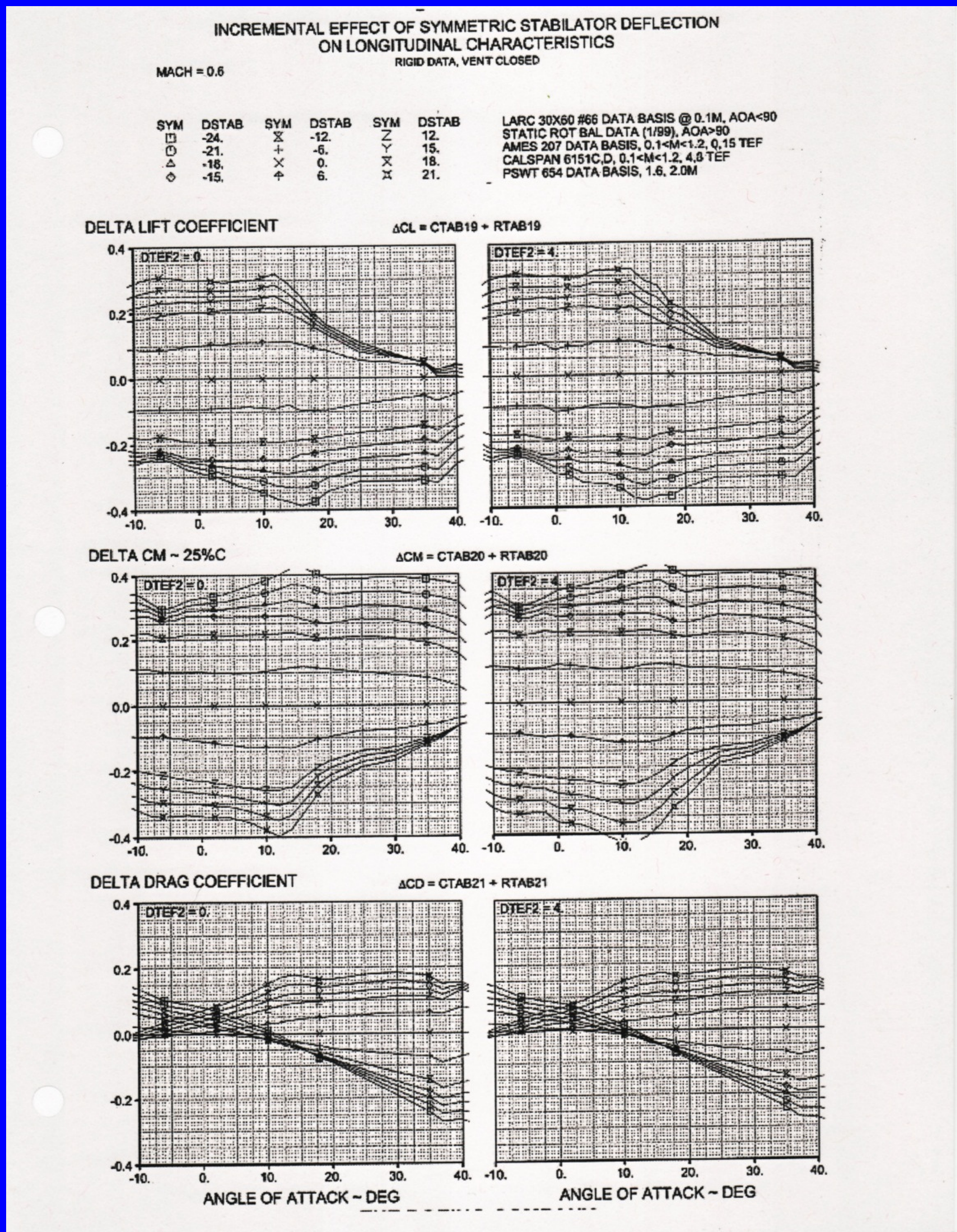
AERO COEFFICIENT CONVENTIONS

- ORIGIN
  - Simplified, linear analytical tool
  - Stability derivatives
- FUNCTIONS OF:
  - Steady state flight conditions
  - Control positions
  - Small perturbations
- FLIGHT SIMULATION NEEDS:
  - Large perturbations
  - f(*most influential variables*)
  - Non-linear, multivariate
  - RESULT: large sets of data points

TYPICAL WIND TUNNEL DATA

T-34C Lift Coefficient





A few of the hundreds of aero data tables in an F-18 simulation model

Effect of stabilator deflection on lift, pitch moment, drag as function of AOA and TEF for one Mach number.

AERO DATA HANDLING

- Data tables *vs* Curve Fitting
- Number of tables
- Size of tables
- Number of variables per table
- Interpolation routines

BASIC FIXED WING AERODYNAMIC COEFFICIENTS
Stability Axis System

Lift Force

CL = CL(AOA) + CLDE*DE + CLDF*DF
(*or CL0+CLAOA*AOA)*

Drag Force

CD = CD(CL) + CDDF*DF + CDDG*DG + CDDSB*DSB

Side Force

CY = CYBETA*BETA + CYP*P + CYR*R

Roll Moment

CRS = [CRDA*DA + CRDR*DR] + CRBETA*BETA + (CRP*P + CRR*R)*B/2V

Pitch Moment

CMS = [CMDE*DE + CMDF+DF] + CM(CL) + (CMQ*Q + CMADOT*ADOT)*C/2V

Yaw Moment

CNS = [CNDR*DR + CNDA*DA] + CNBETA*BETA + (CNR*R + CNP*P)*B/2V

where:

AOA=angle of attack
ADOT=AOA rate
BETA=sideslip
P = roll rate
Q = pitch rate
R = yaw rate

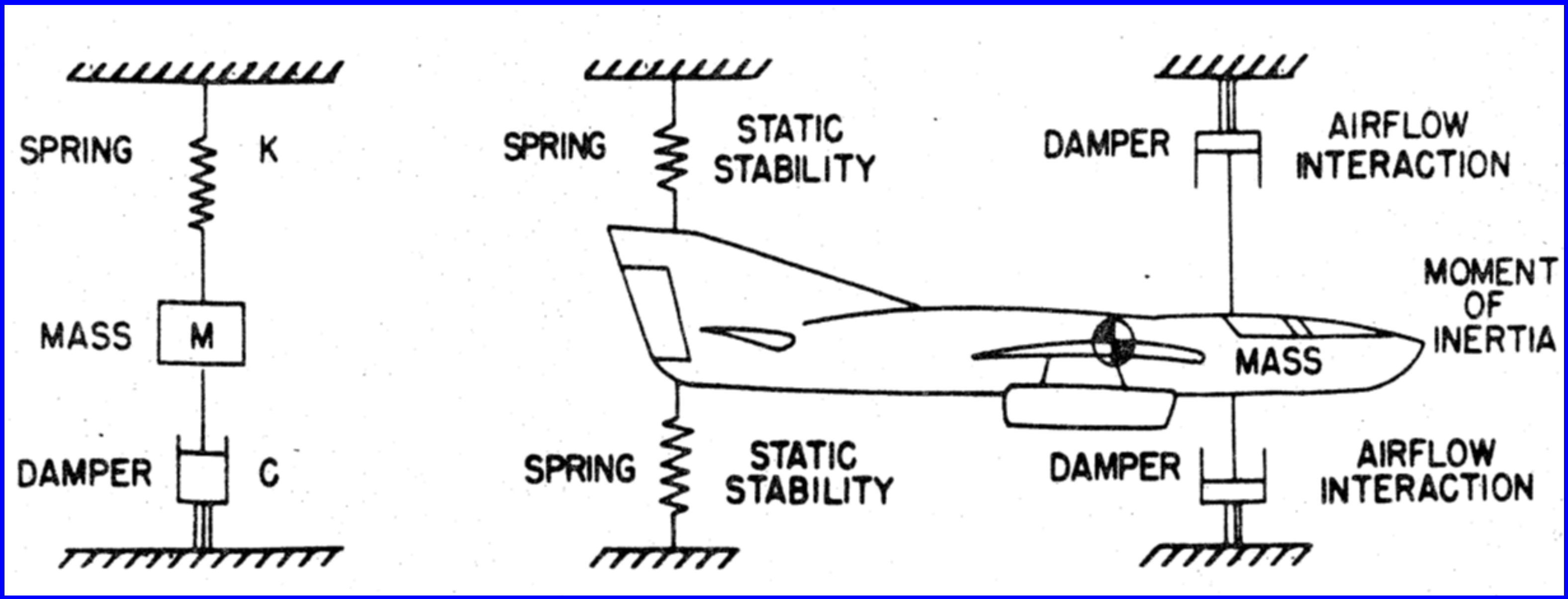
DE=elevator deflection
DA=aileron deflection
DR=rudder deflection
DF=flap deflection
DSB=speed brake deflection
DG=landing gear position

B=wing span
C=mean aero chord
V=true airspeed

AIRPLANE in FLIGHT

analogous to

SPRING-MASS-DAMPER System

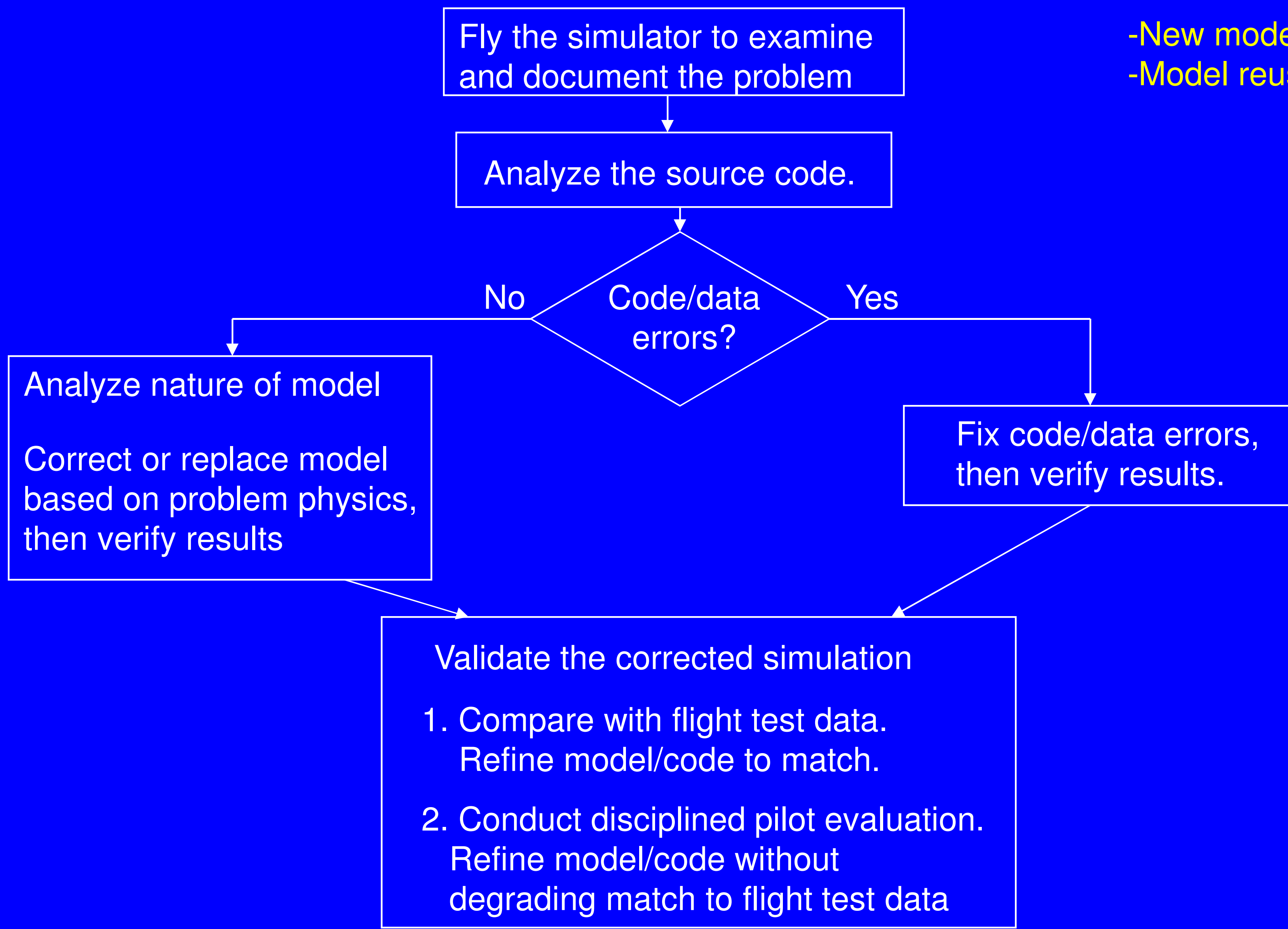


AERO COEFFICIENT CAVEATS

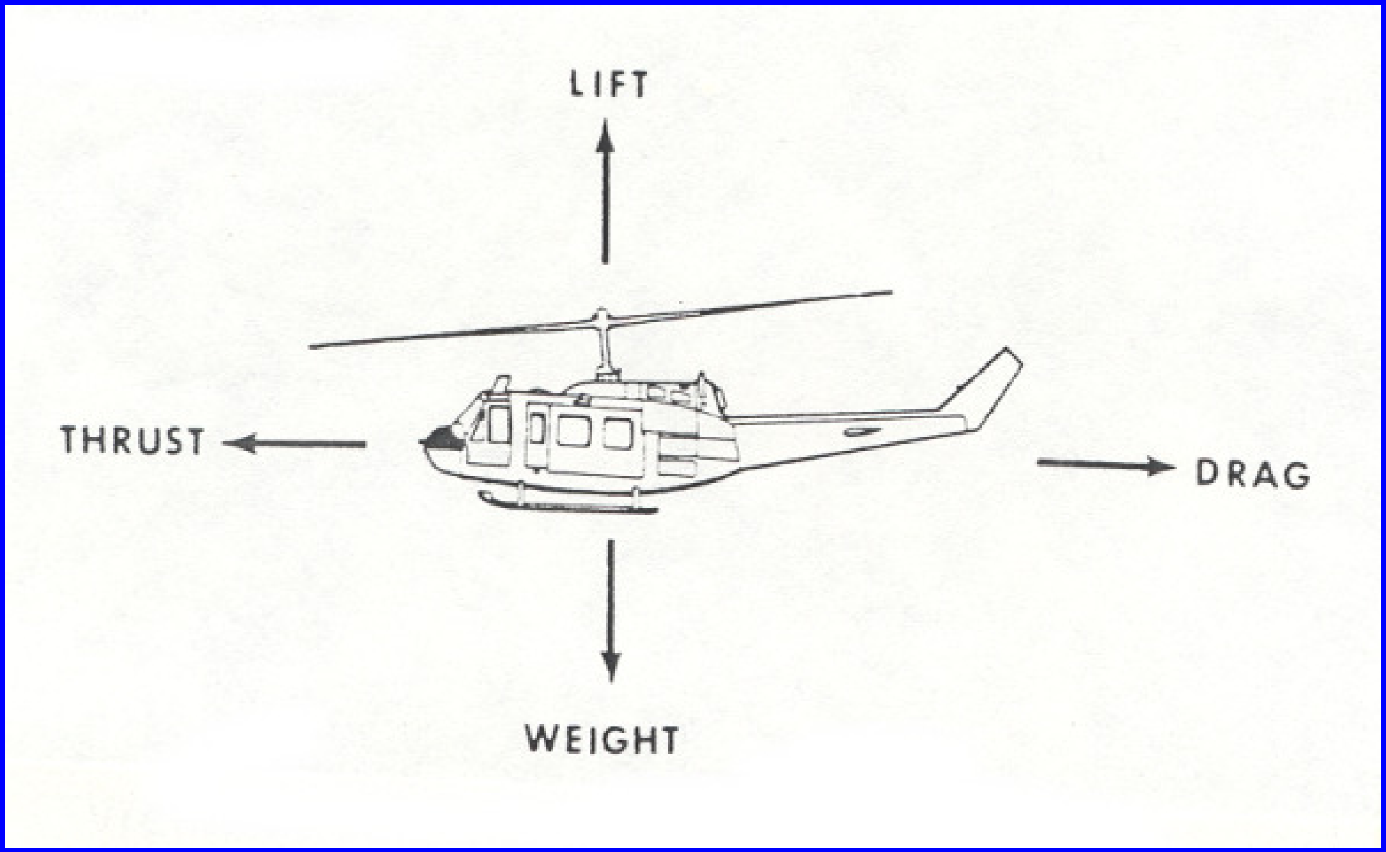
- Non-Universal Format
  - Body or Stability Axis
  - Per degree or per radian
  - Dimensional or Non-dimensional
- Excess Terms or Variables
- Suspect Accuracy
  - Limited flight test validation
  - 5% (lift & drag coefficients)
  - 50% (dynamic coefficients)
- Aero Analyst Skill Requirement
  - Must correlate equation parameters with physical behavior

AERO MODEL PROBLEM RESOLUTION PROCESS

-New models
-Model reuse



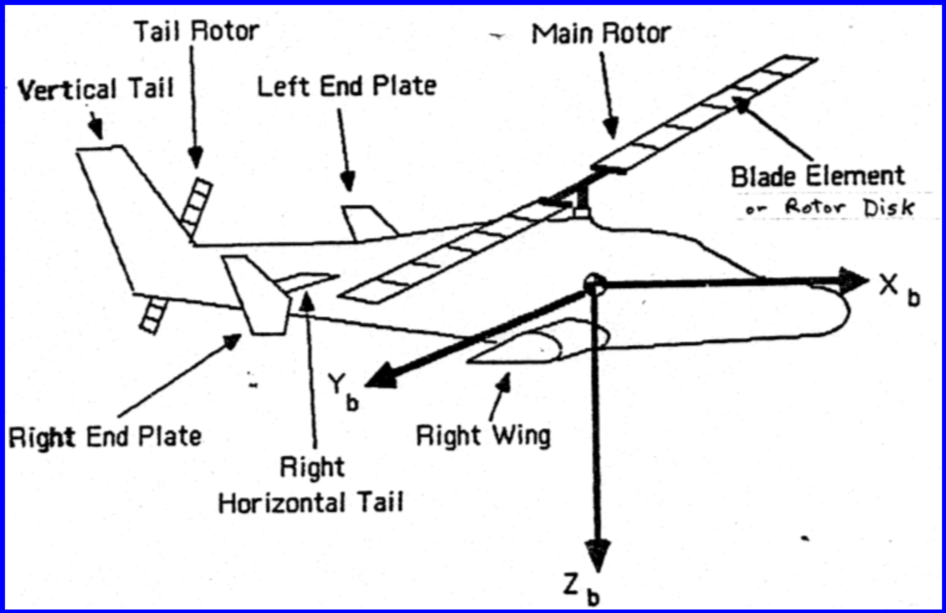
HELICOPTER AERO MODELING



HELICOPTERS

Additional Modeling Complexities

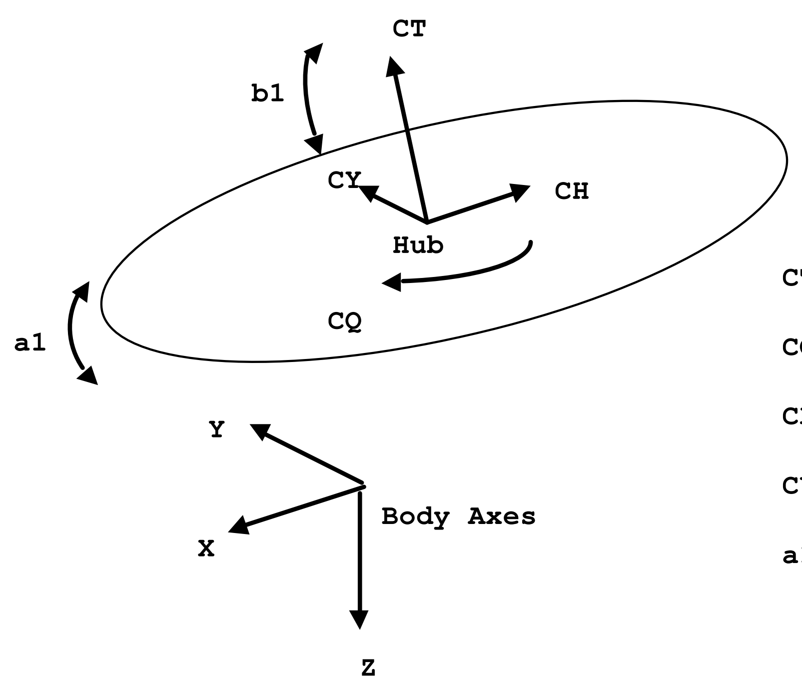
- Blade stall
- Rotor inflow
- Rotor downwash effects
- Ground effects
- Unsteady aerodynamics
- Blade elastic models



ROTOR AERODYNAMIC MODELS

- Perturbation
  - No distinct rotor
- Rotor Disk
  - Bailey, equations
- Rotor Map
  - Link, 3-D data tables
- Blade Element Models
  - True physics

HELICOPTER ROTOR - AERO TERMINOLOGY



CT = Rotor Thrust (Lift)

CQ = Rotor Torque

CH = Rotor Drag Force

CY = Rotor Side Force

a1, b1 = Rotor Flapping Angles

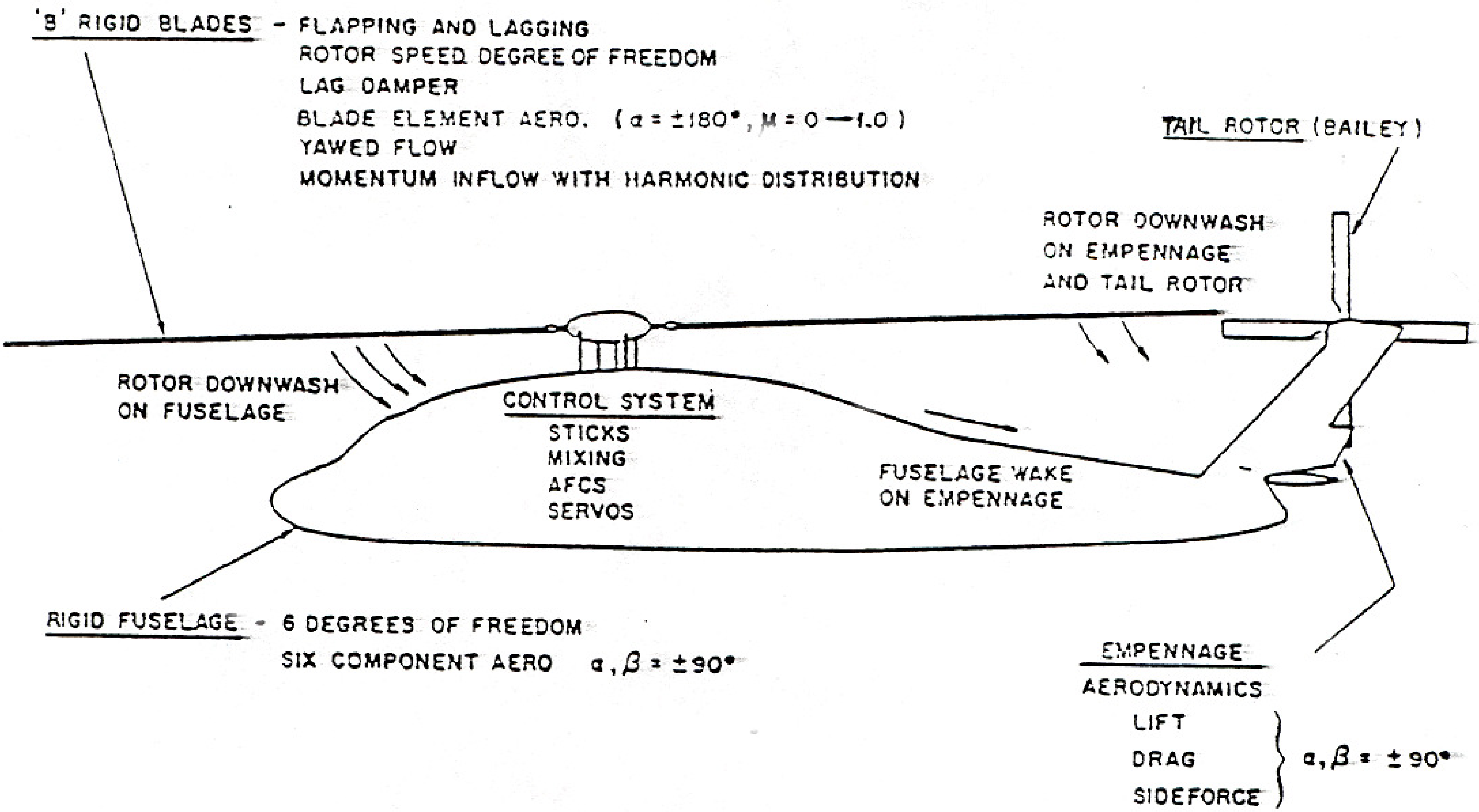
DISK ROTOR MODELS

- Quasi-static assumption
- Common methods used:
  - Rotor map (large data tables)
  - Analytical or Equation
- Advantages
  - General purpose computers
  - Easy to ‘tweek’ for pilot opinion
- Disadvantages
  - Engineering mods difficult
  - Poor off-nominal flight fidelity
  - Limited dynamic accuracy

BLADE ELEMENT ROTOR MODELS

- Several segments per blade
- Compute CL, CD, CM per section
  - Requires rapid, almost parallel processing
  - Sum all segments and blades
- Blade flapping, lead-lag modeled
- Advantages
  - Good dynamic response simulation
  - Direct engineering physics representation
- Disadvantages
  - Expertise required

'GENHEL' MODEL COMPONENTS



BLADE ELEMENT ROTOR MODELS
Successful Examples

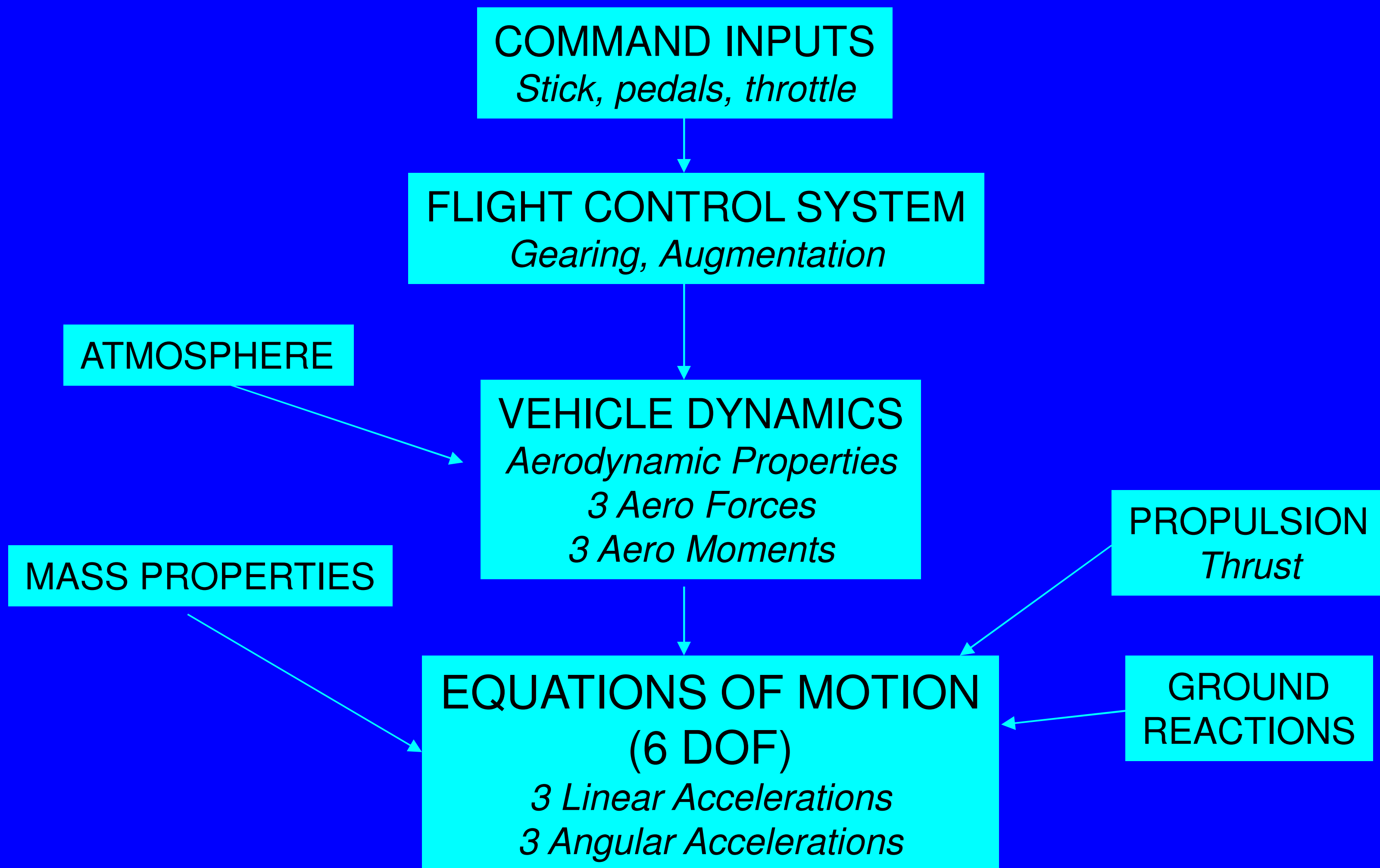
- Implementation (USMC CH-46, CH-53)
  - Blade segments: 6 to 10
  - Azimuth step size: 8 - 10 degrees
  - Validation/De-bug: OK with right expertise
- Challenges
  - Minimize azimuth step size (< 15 deg)
  - Improve dynamic inflow modeling
  - Improve turbulence modeling

Implementation overview for AH-1W WST: See AIAA 2006-6809

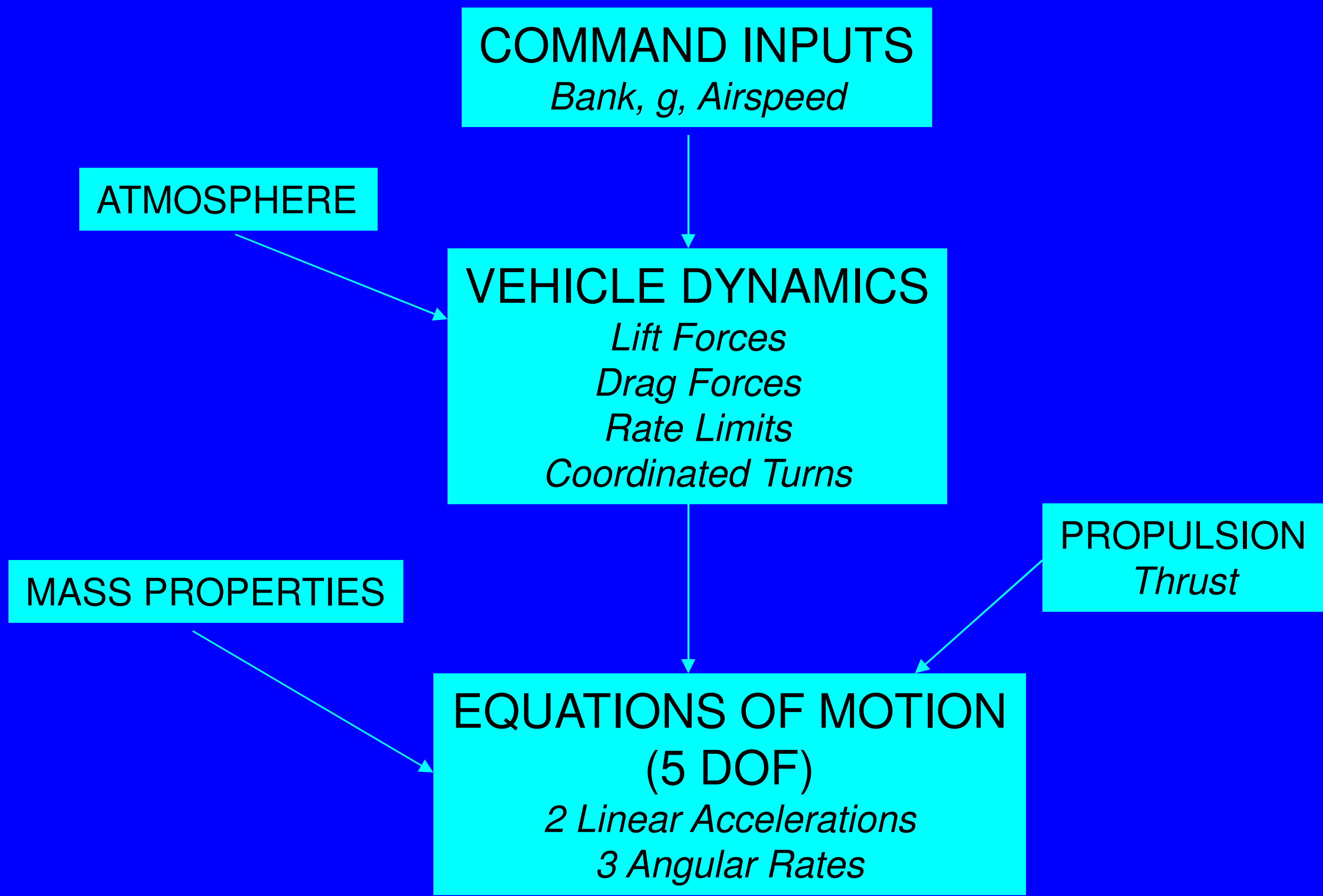
Challenges:
- Ship air wake model
- Variation over rotor

SIMPLIFIED FLIGHT MODELS
&
OTHER COMPONENTS

6 DOF FORCE & MOMENT MODEL



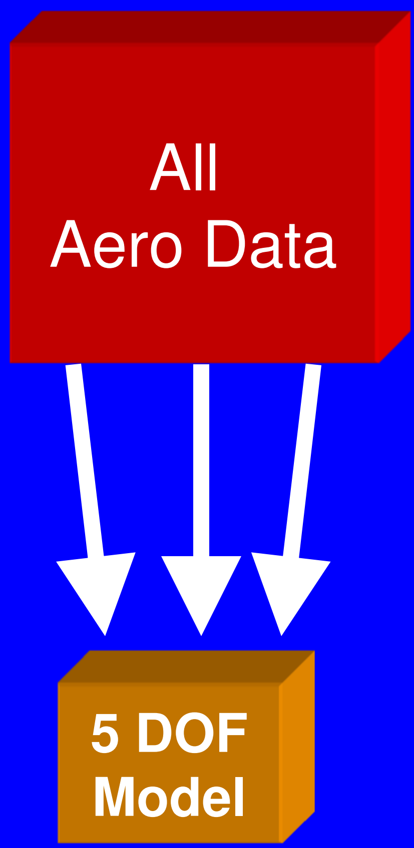
5 DOF PERFORMANCE MODEL



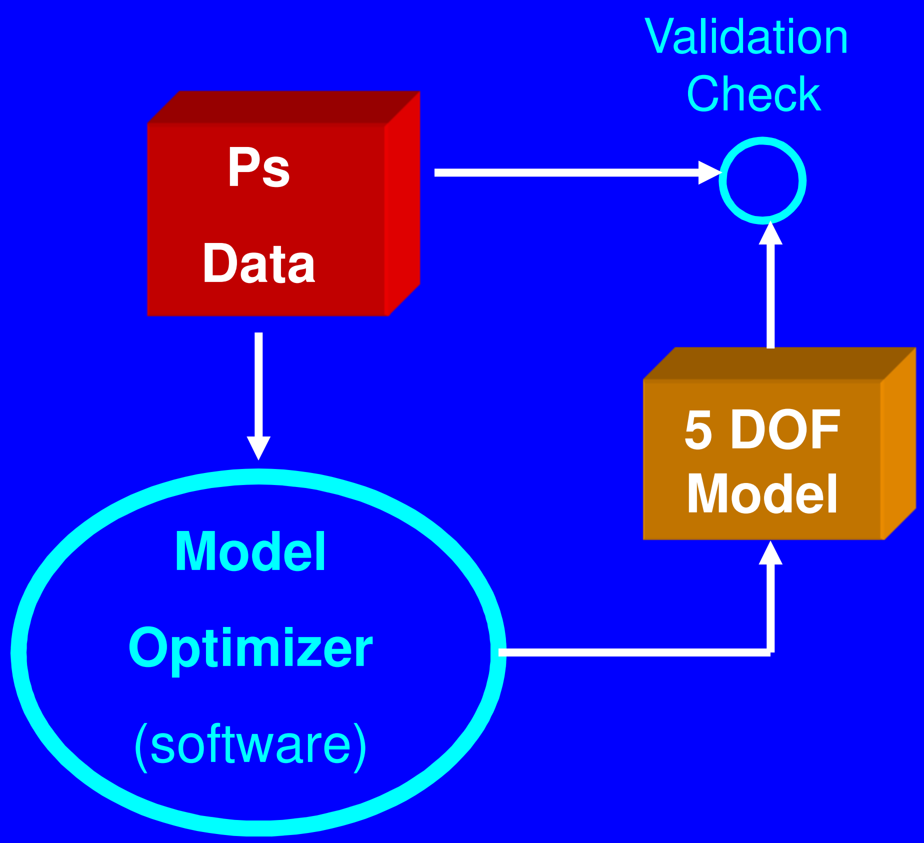
5 DOF Performance Modeling Methods

Anderson & Schab, AIAA 93-3553-CP

Old Method



Self Validating Method



5 DOF RATE MODEL

COMMAND INPUTS
Roll Rate
Pitch Rate
Airspeed

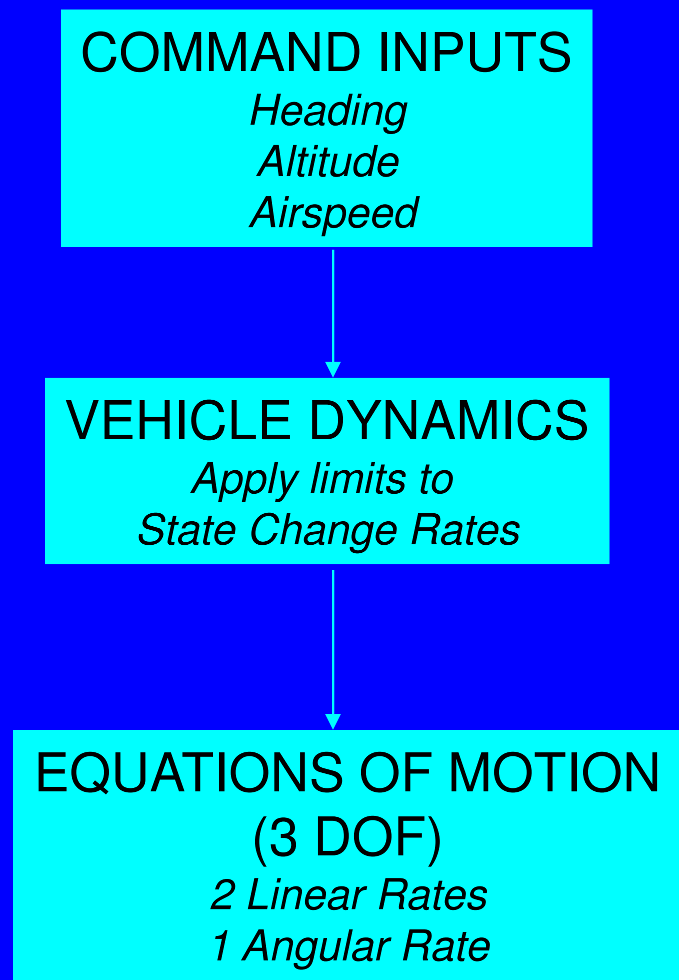
↓

VEHICLE DYNAMICS
Rate Limits
Coordinated Turns

↓

EQUATIONS OF MOTION
(5 DOF)
2 Linear Rates
3 Angular Rates

3 DOF POINT MODEL



ATMOSPHERE SIMULATION

- Basic Parameters
  - Temperature, pressure, density
  - Standard atmosphere references
- Winds, Gusts
- Turbulence, Wind Shear
  - FAA Models: AC 120-141
- Storm Cells
- Air Wake (burble)
  - Ships, aircraft, buildings, land features
  - Aircraft carrier burble models:
    - MIL-F-8785 & MIL-STD-1797

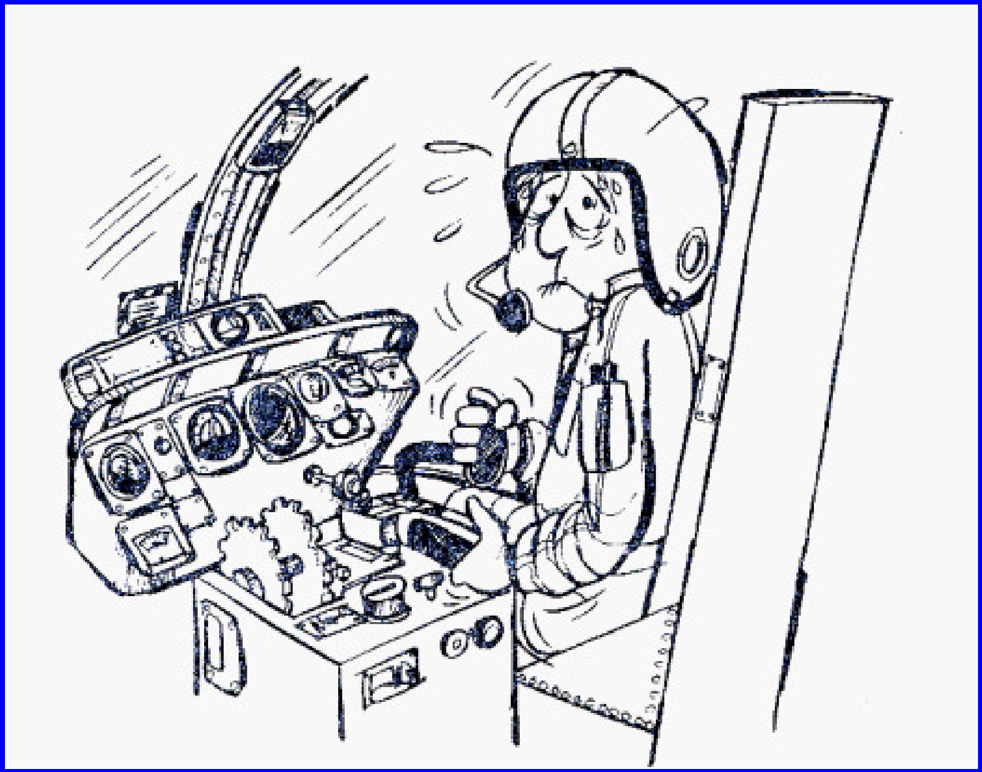
POWERPLANT SIMULATION

- Steady State Performance
  - Match engine manufacturer data
  - Table look-up
- Transient Response
  - Closed loop models
    - Fuel control system
    - Cockpit display parameters

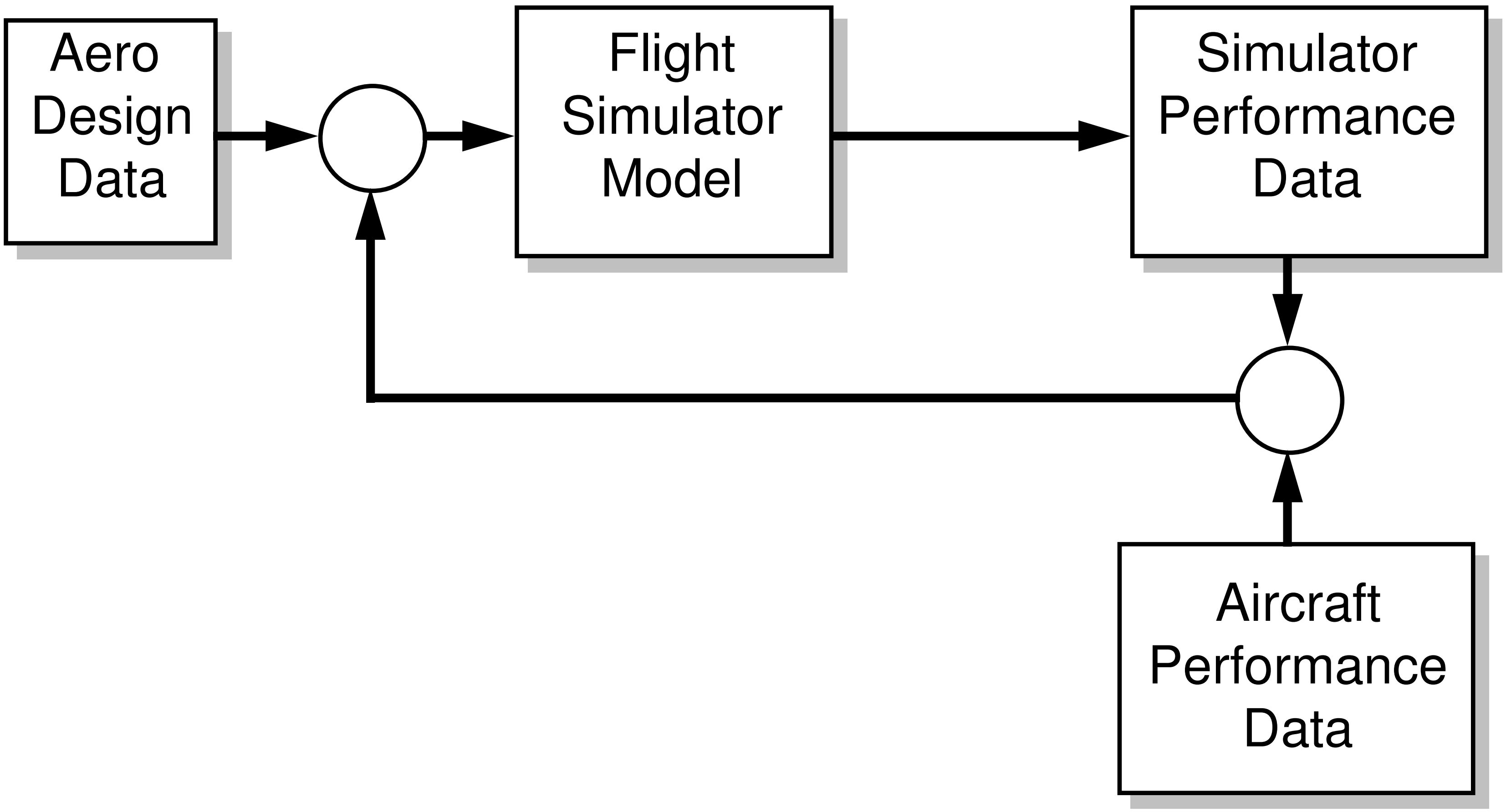
GROUND REACTIONS

- Very Dynamic Phenomenon
  - Landing impact
  - Tilting surfaces (ship, slopes)
  - Surface friction
  - Steering, castoring, suspension
  - Braking
  - Computational stability
- Sparse Data
  - Tire, strut dynamics
  - Pilot performance

FLIGHT TEST DATA



FLIGHT SIMULATION
Modeling & Validation



CLASSICAL FLIGHT TEST METHODS

- Test Purpose
  - Generate <u>repeatable</u> test results
  - Demonstrate mission compliance
  - Document contract guarantees
  - Evaluate pilot workload
- Test Techniques
  - Test Pilot School manuals
- Test Data
  - Substantiate pilot opinion
  - Problem analysis
- Subjective Evaluations
  - Disciplined methodology
- Trained Test Pilots
  - Much more than a ‘Golden Arm’

FLIGHT TEST CATEGORIES

- Performance
  - Airframe lift & drag
  - Powerplant thrust & fuel use
- Flying Qualities
  - Stability & Control Characteristics
  - Open loop test techniques
- Mission Effectiveness
  - Mission related tasks (closed loop testing)
  - Defined rating scales

FLYING QUALITIES TESTS

Results Analysis

- Stability Derivatives
  - FQ results show:
    - manifestation, not a direct measure
  - Correlation requires further analysis

- Separation of axes
  - Longitudinal (x, z, pitch)
  - Lateral-Directional (y, roll, yaw)

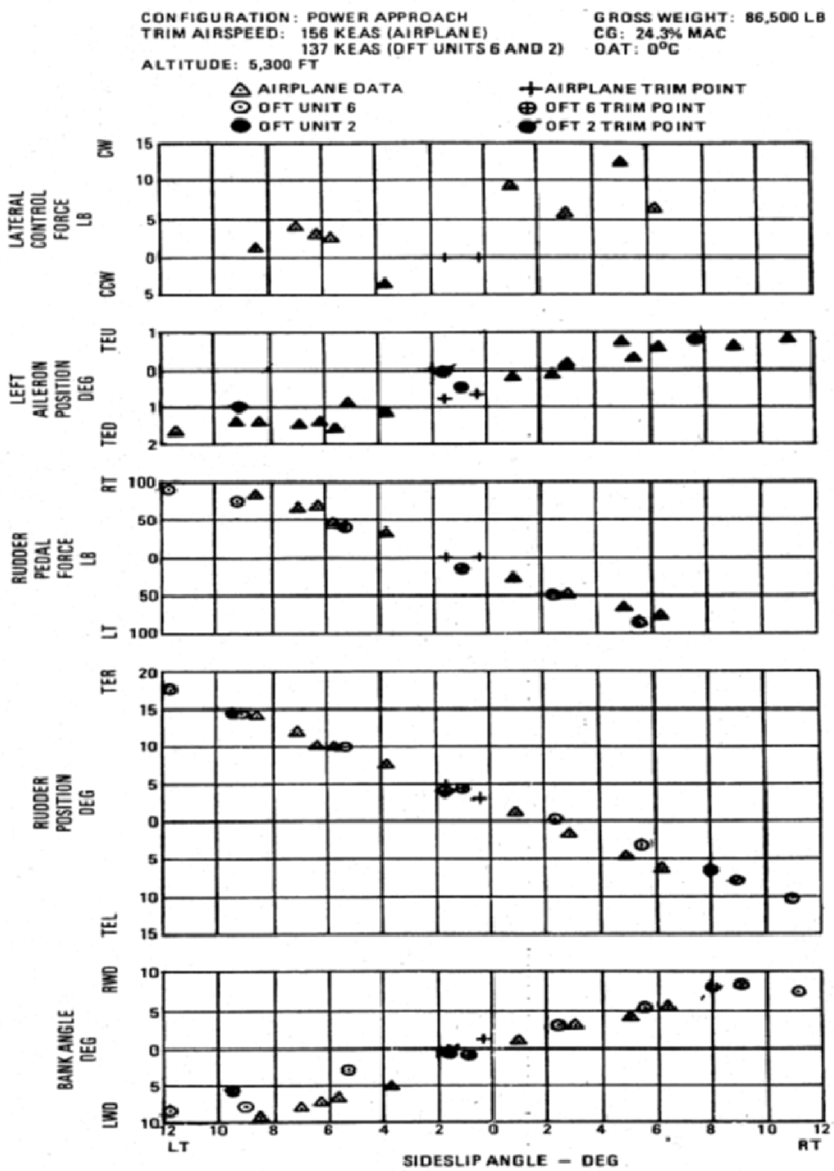
TYPICAL FLIGHT FIDELITY TEST DATA SET

Flying Qualities & Performance

- Flight control system
  - Mechanical characteristics
  - AFCS
- Takeoff / landing FQ&P
- Static trim points & cruise performance
- Level acceleration/decelerations
- Climb performance
- Level turning performance
- Trim changes with configuration changes
- Longitudinal stability (static, maneuvering, & dynamic)
- Lateral-directional stability (static & dynamic)
- Lateral control effectiveness
- Stall / high AOA characteristics
- Asymmetric power FQ&P
- Mission tasks

FLIGHT TEST CRITERIA DATA UNIQUE TO ROTORCRAFT

- Engine governing characteristics
- Slow airspeed FQ&P
  - Sideward, rearward, forward flight
  - Critical azimuth
- In-flight performance
  - Hover
  - Vertical climb
- Control response (4 axes)
- Rotor characteristics
  - Autorotation
  - Blade stall
  - Power settling
  - Vibration



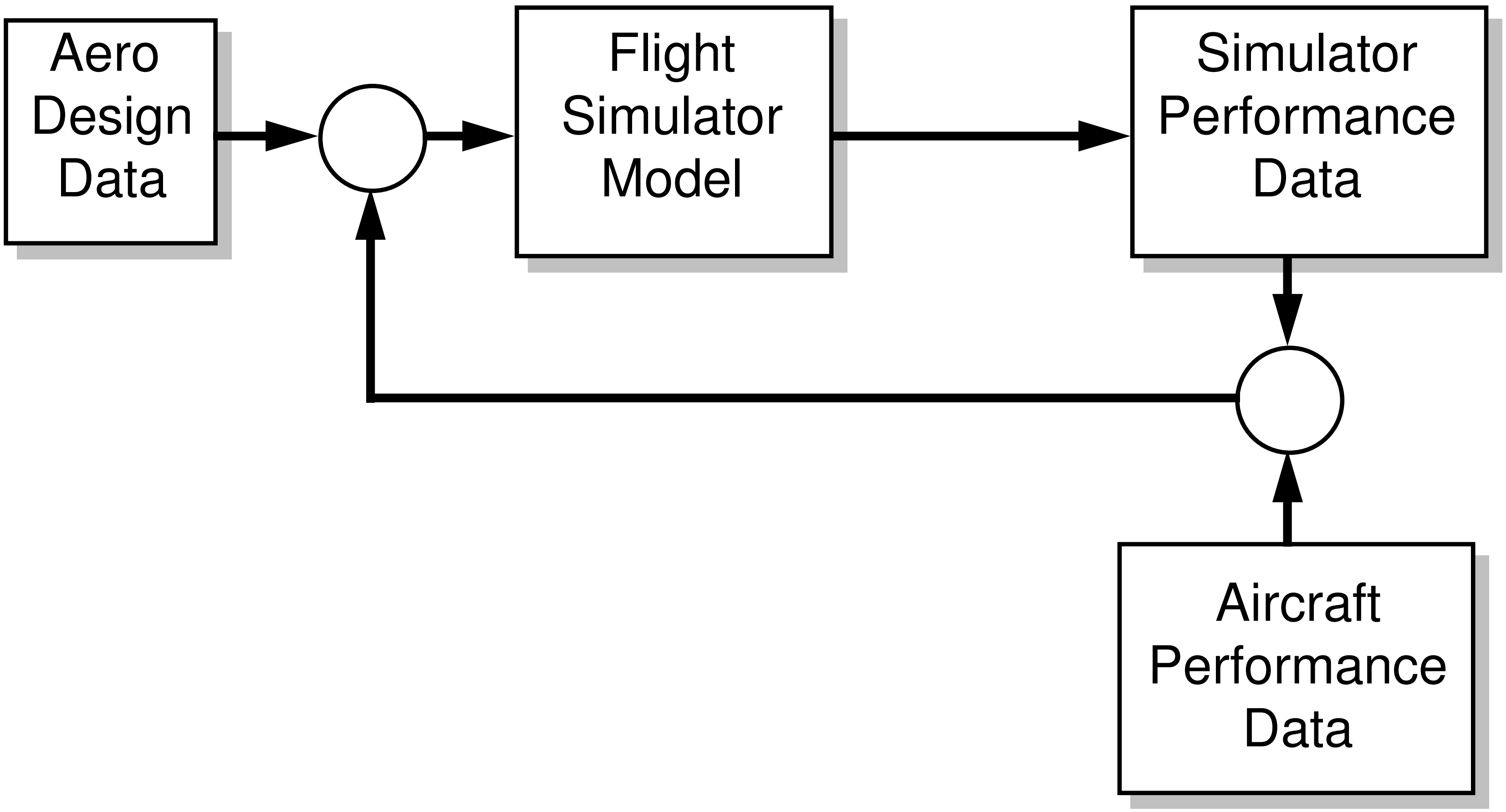
STATIC LATERAL-DIRECTIONAL STABILITY
(Configuration PA)
P-3C Airplane, BuNo 160290
and
P-3C OFT, Device 2F87(F) (Units 6 and 2)

Typical Flight Test Data

Note:
- Test conditions info
- Data source info
- Data scatter!

Skilled analysis required!

FLIGHT SIMULATION
Modeling & Validation



FLIGHT SIMULATOR MODELING DECISIONS
(Type, Complexity, Flexibility)

Example: AV-8B Weapon Systems Trainer

Requirements:
- Pilot Skills
- Air to Air Tactics
- Air to Ground Tactics
- Incident Investigation

OWNSHIP (AV-8B) MODELS
- Aero, Engine, Flight Controls
- All aircraft systems
- Weapons sensors, fly-out, ballistics

FLIGHT ENVIRONMENT
- Atmosphere, turbulence, winds
- Moving models (Ship landings, tankers, formation)

TACTICAL PLAYERS
- Threat Aircraft (Dynamics, Behavior, Weapons)
- SAM, AAA (Dynamics, Behavior)
- Surface / Ground Targets (Dynamics, Behavior)
- *USER EDITING CAPABILITY*

MULTIPLE COCKPITS
- Dual Dome
- Network

Reference Resources

- Galloway Course Notes: Mathematical Modeling, Binghamton University Short Course Notes.
- Allerton, D., “Principles of Flight Simulation,” AIAA, 2009 .
- Anderson J.D., "Introduction to Flight", McGraw-Hill, New York, 2000.
- ANSI/AIAA - American National Standard - Recommended Practice: "Atmospheric and Space Flight Vehicle Coordinate Systems", ANSI/AIAA R-004-1992, February 1992.
- Dreier, M.E., “Introduction to Helicopter and Tiltrotor Flight Simulation,” AIAA, 2007.
- Howlett J.J., “UH-60A Black Hawk Engineering Simulation Program: Volume I - Mathematical Model,” NASA CR 166309, Dec 1981 ** The classic GENHEL blade element model*
- Creech, B., Hildreth, B., “Adjusting a Helicopter Rotor Blade Element Model to Match Sparse Criteria Data”, AIAA paper 2006-6809, AIAA M&S Technologies Conference, Keystone, CO, August 2006.
- Prouty R.W., "Helicopter Performance, Stability, and Control," Robert E. Krieger Publishing Co., Inc., Malabar, FL, 1990 .
- Roskam J., “Airplane Flight Dynamics and Automatic Flight Controls,” Roskam Aviation and Engineering Corp., Lawrence, KA, 1979.
- Stevens B.L., Lewis F.L., "Aircraft Control and Simulation", 2nd Edition, John Wiley and Sons, New York, 2003).
  - **(First 3 chapters: cohesive build-up of real time flight simulation methods-EOM, aircraft modeling, computation tools.)*
- USN & USAF Test Pilot School Manuals