SPIHT Algorithm

$\mathcal{O}$(i,j): set of coordinates of all offspring of node (i,j); *children only*
$\mathcal{D}$ (i,j): set of coordinates of all descendants of node (i,j); *children, grandchildren, great-grand, etc.*
$\mathcal{H}$ (i,j): set of all tree roots (nodes in the highest pyramid level); *parents*
$\mathcal{L}$ (i,j): $\mathcal{D}$ (i,j) – $\mathcal{O}$(i,j) (all descendents except the offspring); *grandchildren, great-grand, etc.*

Initialization

n = $\lfloor \log_2 (\max |\text{coeff}|) \rfloor$
LIP = All elements in $\mathcal{H}$
LSP = Empty
LIS = $\mathcal{D}$'s of Roots

Significance Map Encoding ("Sorting Pass")

Process LIP

for each coeff (i,j) in LIP
Output S_n(i,j)
If S_n(i,j)=1
Output sign of coeff(i,j): 0/1 = -/+
Move (i,j) to the LSP
Endif
End loop over LIP

$$S_n(\Gamma) = \begin{cases} 1, & \max_{(i,j)\in\Gamma} \{|c_{ij}|\} \geq 2^n \\ 0, & \text{otherwise} \end{cases}$$

SPIHT Algorithm (cont.)

Process LIS

for each set (i,j) in LIS

if type *D*

Send $S_n(D(i,j))$

If $S_n(D(i,j))=1$

for each $(k,l) \in O(i,j)$

output $S_n(k,l)$

if $S_n(k,l)=1$, then add (k,l) to the LSP and output sign of coeff: 0/1 = -/+

if $S_n(k,l)=0$, then add (k,l) to the end of the LIP

endfor

endif

else (type *L*)

Send Sn(*L*(i,j))

If Sn(*L*(i,j))=1

add each $(k,l) \in O(i,j)$ to the end of the LIS as an entry of type *D*

remove (i,j) from the LIS

end if on type

End loop over LIS

Refinement Pass

Process LSP

for each element (i,j) in LSP – *except* those just added above

Output the nth most significant bit of coeff

End loop over LSP

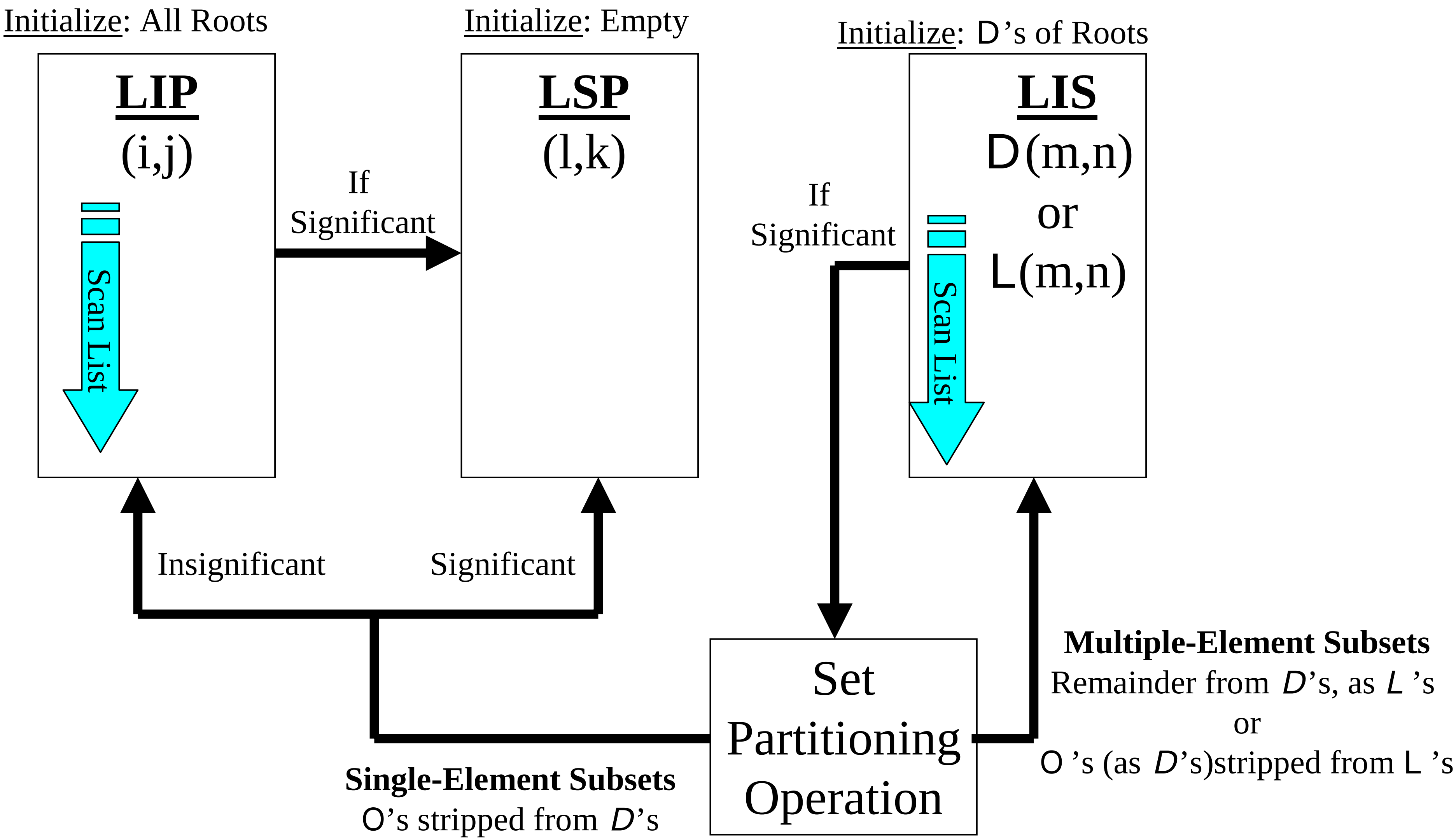
Update

Decrement n by 1

Go to Significance Map Encoding Step

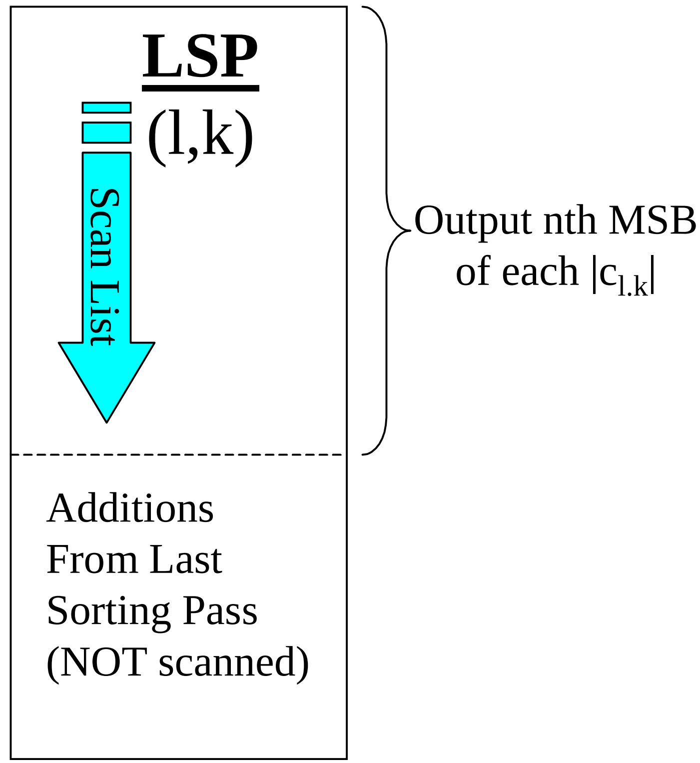
Adaptive Arithmetic Code (Optional)

SPIHT Sorting Pass



SPIHT Refinement Pass

Initialize: Empty



Example of SPIHT

From Kahlid Sayood's Book

26	6	13	10
-7	7	6	4
4	-4	4	-3
2	-2	-2	0

Initialization

LIP

(0,0) $\rightarrow$ 26
(0,1) $\rightarrow$ 6
(1,0) $\rightarrow$ -7
(1,1) $\rightarrow$ 7

LSP

Empty

LIS

(0,1) D $\longrightarrow$ {13, 10, 6, 4}
(1,0) D $\longrightarrow$ {4, -4, 2, -2}
(1,1) D $\longrightarrow$ {4, -3, -2, 0}

$$n = \lfloor \log_2 (26) \rfloor = 4$$

Threshold = 16

After First Sorting Pass

LIP

(0,1) → 6
(1,0) → -7
(1,1) → 7

LSP

Empty

(0,0) → 26

No Refinement Needed

LIS

(0,1) *D* → {13, 10, 6, 4}
(1,0) *D* → {4, -4, 2, -2}
(1,1) *D* → {4, -3, -2, 0}

1 1	0 0 0
Sig./+	Insig.

0 0 0
All *D* sets Insig.

After First Refinement Pass

LIP

(0,1) → 6
(1,0) → -7
(1,1) → 7

LSP

(0,0) → 26

LIS

Significant

(0,1) *D* → {13, 10, 6, 4}
(1,0) *D* → {4, -4, 2, -2}
(1,1) *D* → {4, -3, -2, 0}

n = 3; Threshold = 8

During Second Sorting Pass

LIP

(0,1) → 6
(1,0) → -7
(1,1) → 7
(1,2) → 6
(1,3) → 4

LSP

(0,0) → 26

(0,2) → 13
(0,3) → 10

LIS

Significant Set

(0,1) *D* → {13, 10, 6, 4}
(1,0) *D* → {4, -4, 2, -2}
(1,1) *D* → {4, -3, -2, 0}

Significant Pixels

Insignificant Pixels

(0,1) *D* → {13, 10, 6, 4}

0 0 0
Insig.

1 11 11 0 0

After Second Sorting Pass

LIP

(0,1) → 6
(1,0) → -7
(1,1) → 7
(1,2) → 6
(1,3) → 4

LSP

(0,0) → 26

(0,2) → 13
(0,3) → 10

Refine This

LIS

(1,0) *D* → {4, -4, 2, -2}
(1,1) *D* → {4, -3, -2, 0}

n^{th} MSB of 26 = 1

$+26_{10} = 11\mathbf{1}010_2$
↑ sign bit

Threshold = 4

During Third Sorting Pass

LIP

(0,1) $\rightarrow$ 6
(1,0) $\rightarrow$ -7
(1,1) $\rightarrow$ 7
(1,2) $\rightarrow$ 6
(1,3) $\rightarrow$ 4

(3,0) $\rightarrow$ 2
(3,1) $\rightarrow$ -2
(2,3) $\rightarrow$ -3
(3,2) $\rightarrow$ -2
(3,3) $\rightarrow$ 0

Significant

LSP

(0,0) $\rightarrow$ 26
(0,2) $\rightarrow$ 13
(0,3) $\rightarrow$ 10

(0,1) $\rightarrow$ 6
(1,0) $\rightarrow$ -7
(1,1) $\rightarrow$ 7
(1,2) $\rightarrow$ 6
(1,3) $\rightarrow$ 4

(2,0) $\rightarrow$ 4
(2,1) $\rightarrow$ -4

(2,2) $\rightarrow$ 4

LIS

(1,0) *D* $\rightarrow$ {4, -4, 2, -2}
(1,1) *D* $\rightarrow$ {4, -3, -2, 0}

Both
Sets
Significant

(1,0) *D* $\rightarrow$ {4, -4, 2, -2}
(1,1) *D* $\rightarrow$ {4, -3, -2, 0}

Threshold = 4

After Third Sorting Pass

LIP

$(3,0) \rightarrow 2$
$(3,1) \rightarrow -2$
$(2,3) \rightarrow -3$
$(3,2) \rightarrow -2$
$(3,3) \rightarrow 0$

LSP

$(0,0) \rightarrow 26$
$(0,2) \rightarrow 13$
$(0,3) \rightarrow 10$

$(0,1) \rightarrow 6$
$(1,0) \rightarrow -7$
$(1,1) \rightarrow 7$
$(1,2) \rightarrow 6$
$(1,3) \rightarrow 4$

$(2,0) \rightarrow 4$
$(2,1) \rightarrow -4$
$(2,2) \rightarrow 4$

LIS

Empty