BINGHAMTON UNIVERSITY
State University of New York

EEO 401
Digital Signal Processing

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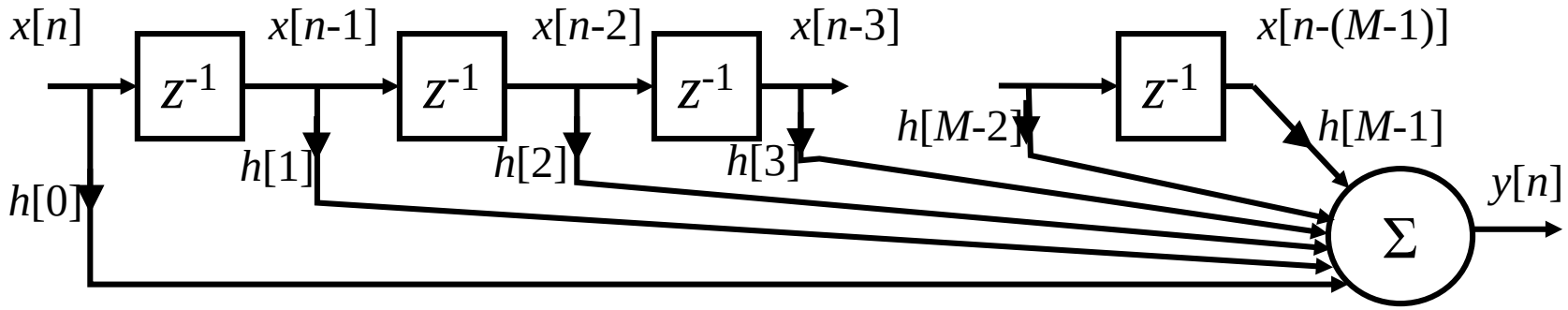
Note Set #5

- Difference Equations – Implementation of DT Systems
- Reading Assignment: Sect. 2.5 of Proakis & Manolakis

Here we will take a first look at how to implement (i.e., build) DT systems… we'll look at this in much more detail later but what we see here will be our foundation.

We've already seen the following structure for an FIR filter based on interpreting its convolution equation in terms of block diagram elements:



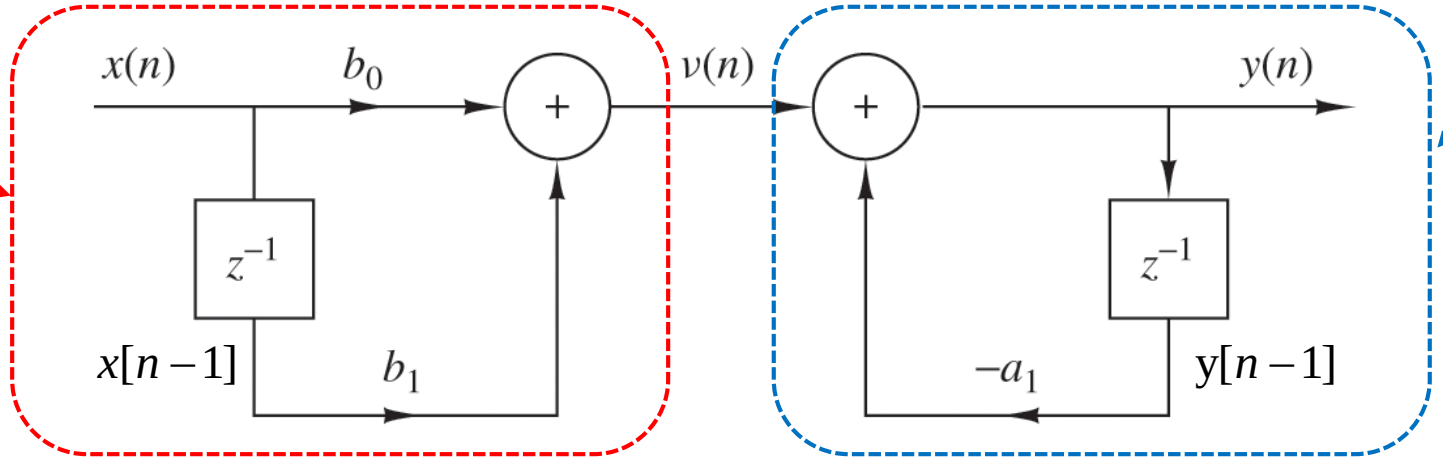
Now we want to look at how to do a similar thing for a DT system described by a general difference equation. We'll start by motivating this with a simple first-order difference equation.

Block Diagrams for First-Order System

$$y[n] = -a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

From this we see that we can use one delay to get $y[n\text{-}1]$ and a second delay to get $x[n\text{-}1]$.

Then we add the various terms together to create $y[n]$



This form… that has separate delays for the input and for the output…. is called **Direct-Form I**

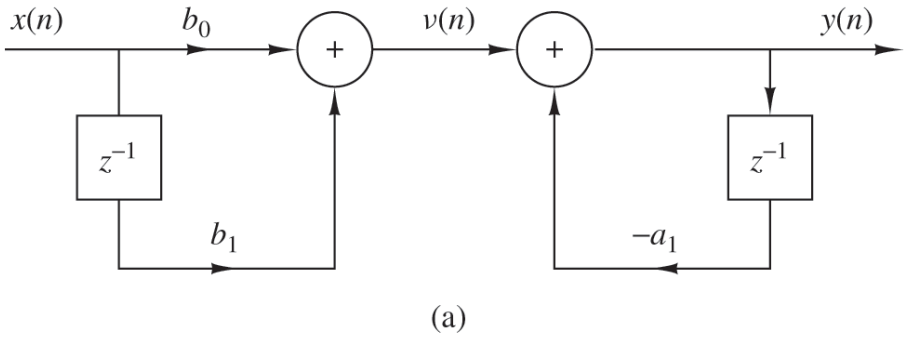
It is possible to reduce the number of delays with a "trick".

Remember… each delay is essentially a memory location so this will reduce the HW needed

Trick to Get Direct-Form II – which has reduced number of delays

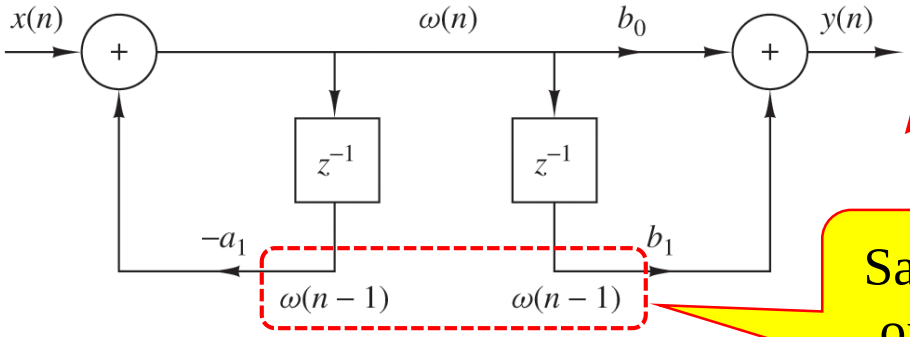
For LTI systems we can interchange their order without changing their overall mathematical result. So…

Direct-Form I



(a)

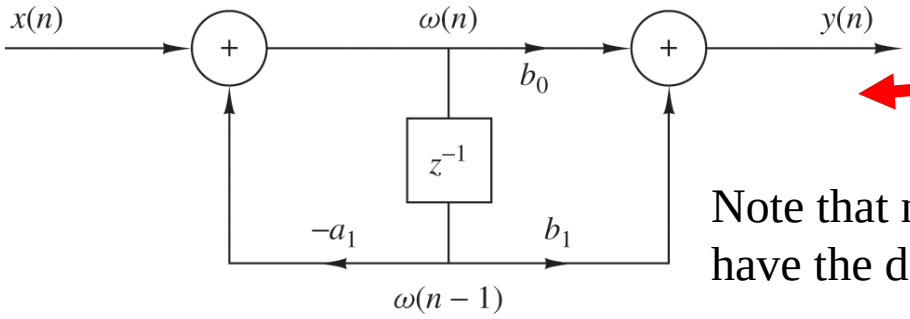
Change order



(b)

Same value… so only one delay is needed!

Direct-Form II

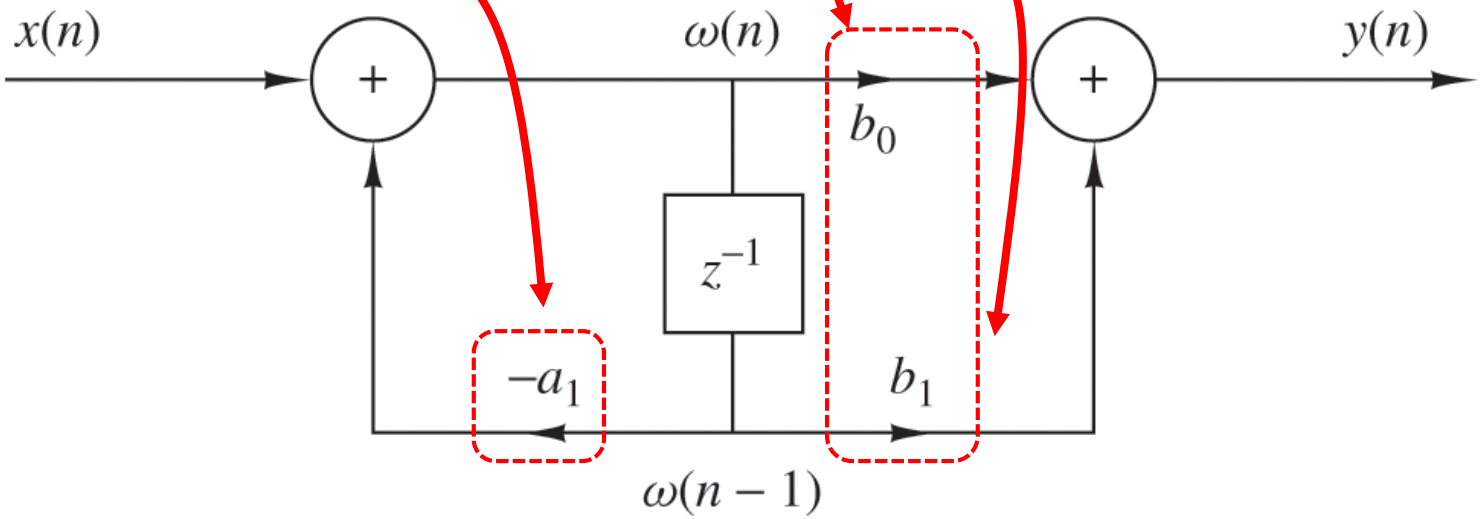


(c)

Note that nowhere in this do we have the delays of input or output!

We can now deduce the structure for a Direct Form II implementation

$$y[n] = -a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

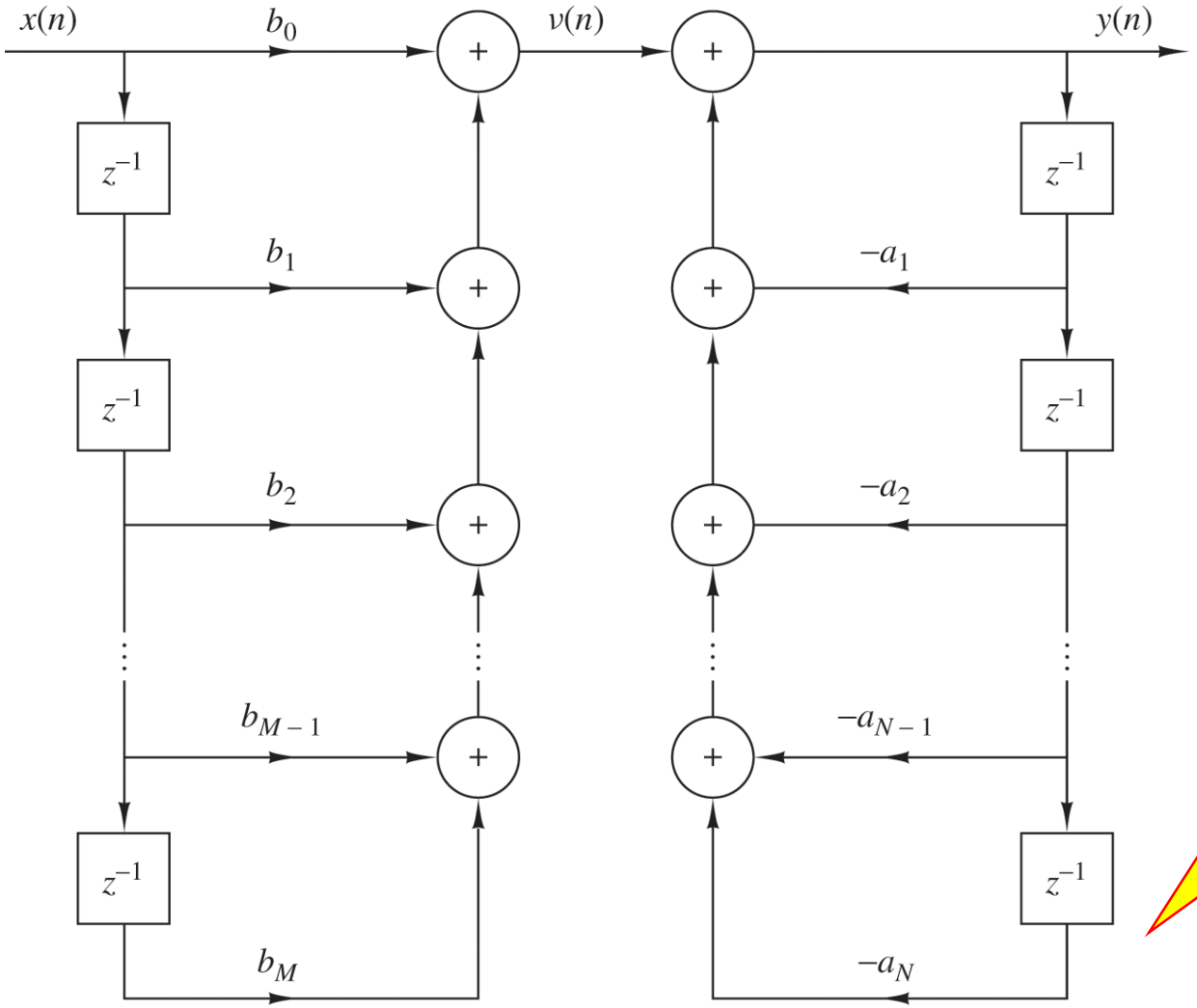


(c)

Block Diagrams for Higher-Order System

We can do the exact same process for a higher-order Difference Eq:

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

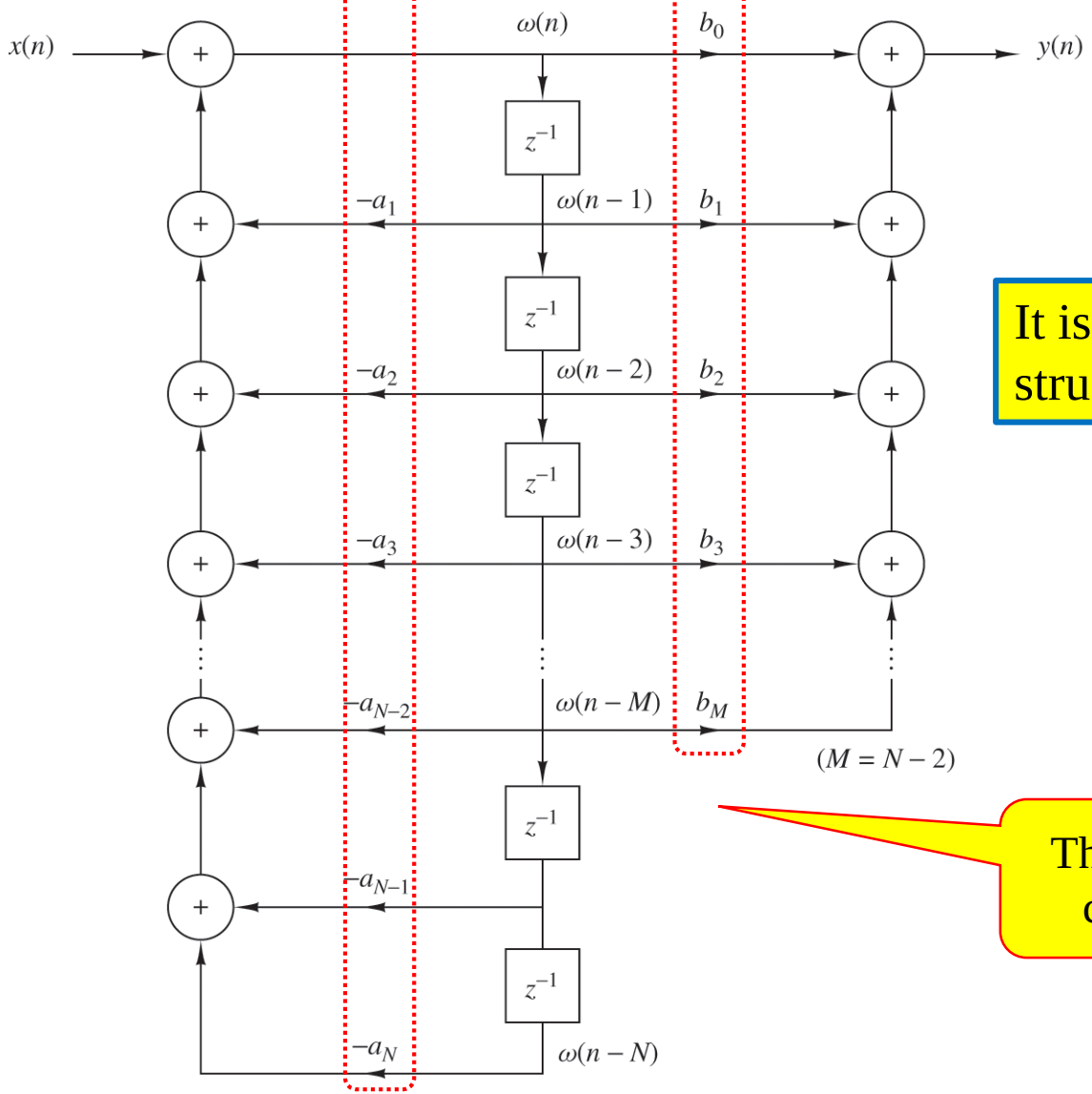


Direct-Form I

Note: *M* and *N* may not be equal… so different # of delays in each side

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

We again use the "trick" to interchange order of delay "stacks"



Direct-Form II

It is easy to determine the structure by inspection!

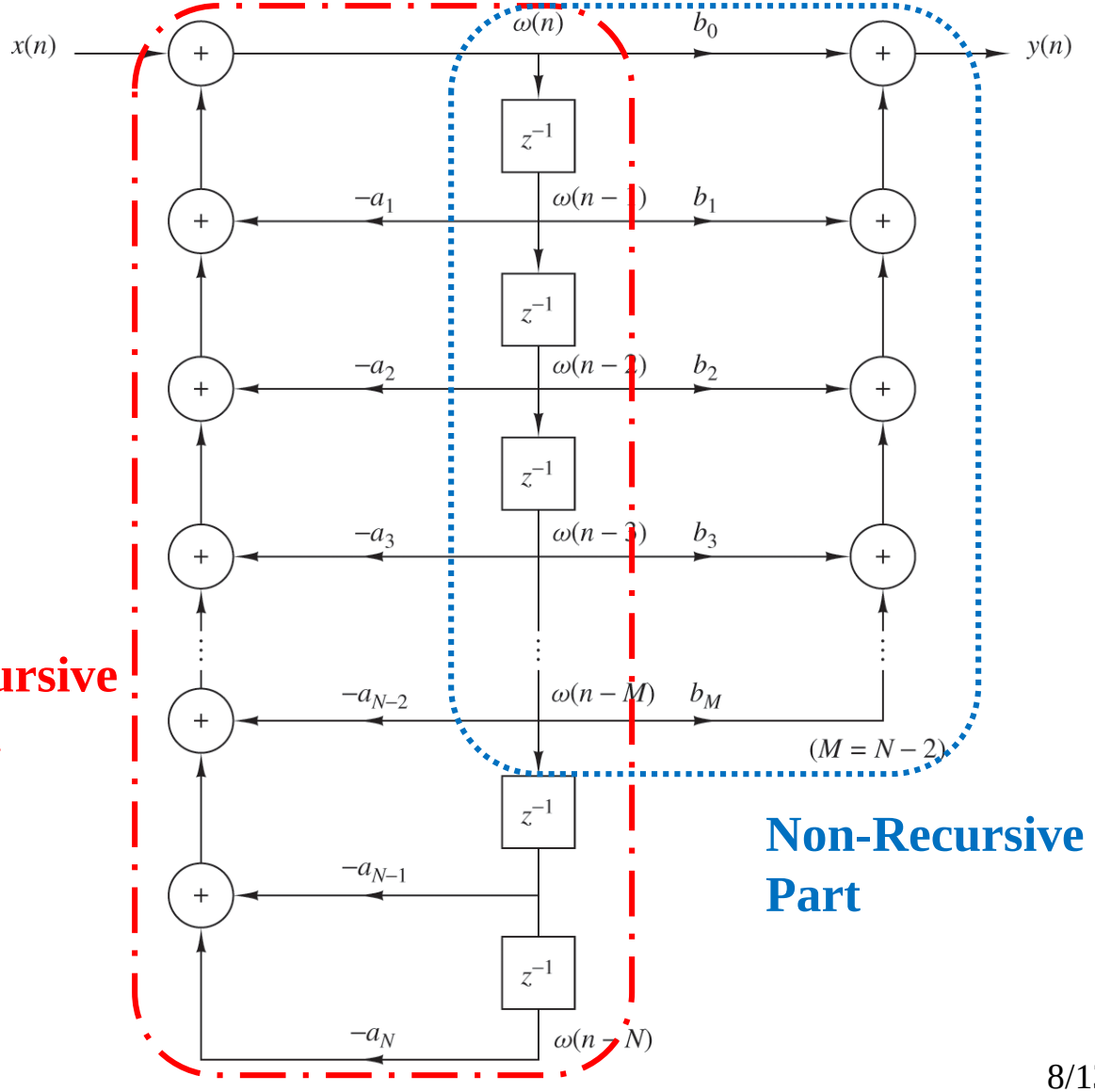
This figure shows the case where $N > M$

Recursive vs Non-Recursive

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

Note that the a_k coefficients are responsible for imparting recursion (feeding back past outputs…).

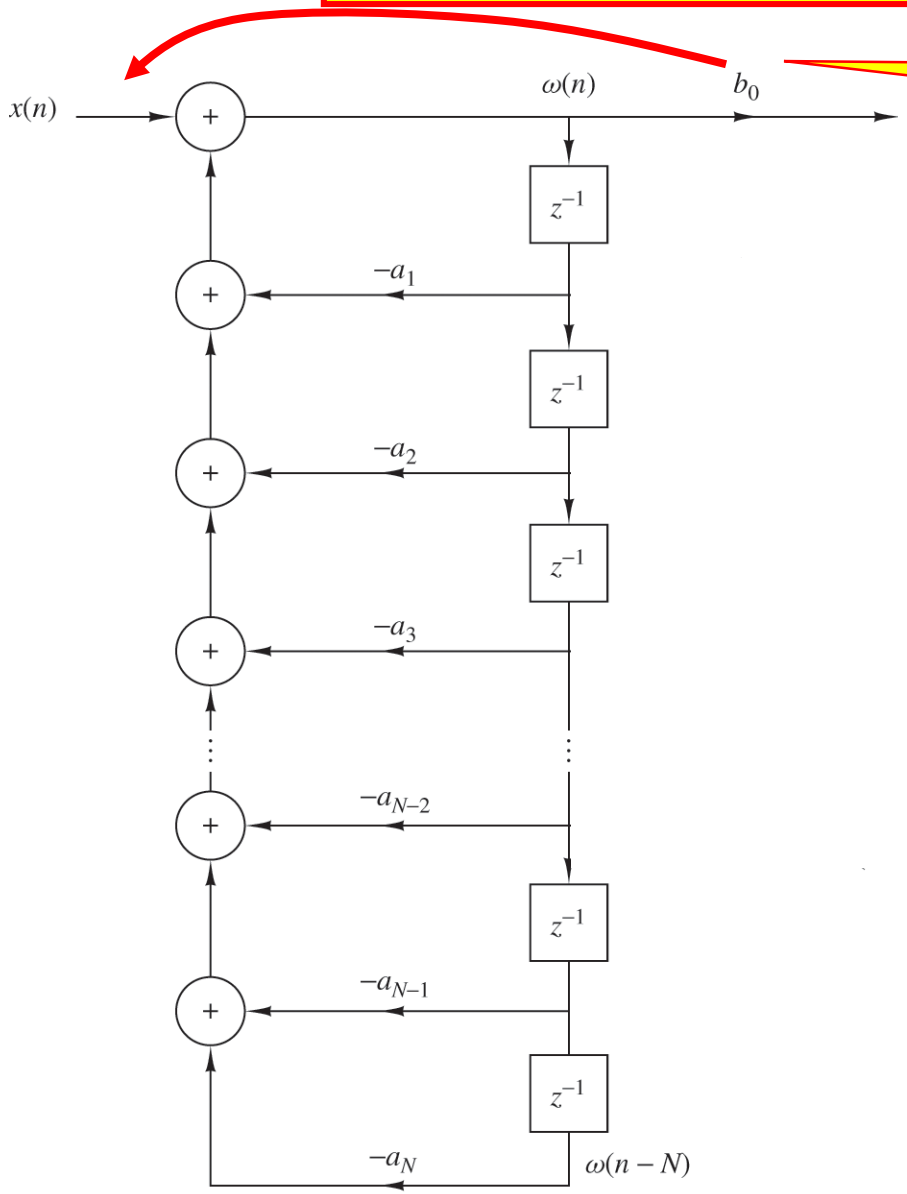
The b_k coefficients do not impart any recursive nature… they constitute the non-recursive part of the sysgtem



Purely Recursive

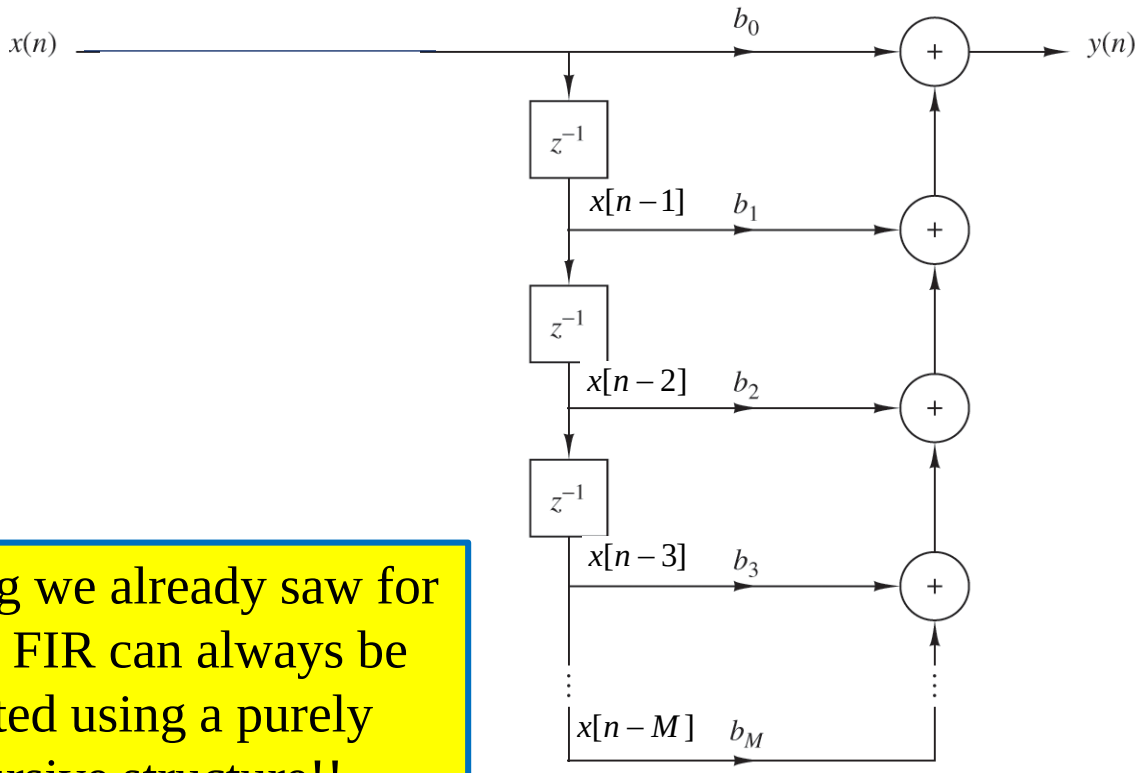
$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + b_0 x[n]$$

Can move this to front…



Purely Non-Recursive ("Moving Average")

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$



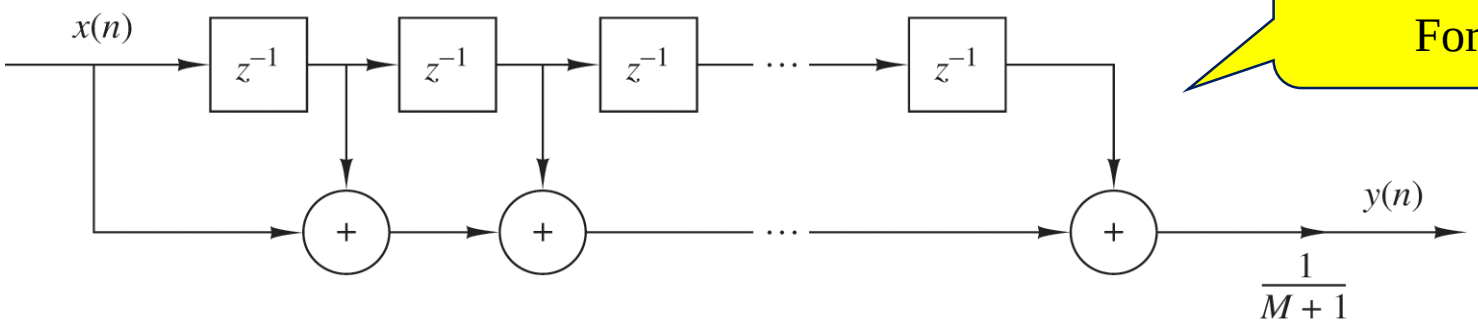
Same thing we already saw for FIR…. So FIR can always be implemented using a purely NON-recursive structure!!

But… any FIR system *can* be implemented using recursion (though not purely recursive).

Example of FIR Implemented using Recursion

$$y[n] = \frac{1}{M+1}\sum_{k=0}^{M} x[n-k] \qquad \text{i.e.,} \quad b_k = \frac{1}{M+1}$$

$n, n-1, \ldots, n-M$



Nonrecursive Form

Can rewrite this system equation as

$$y[n] = \left[\frac{1}{M+1}\sum_{k=0}^{M} x[n-1-k]\right] + \frac{1}{M+1}\left(x[n] - x[n-(1+M)]\right)$$

$n-1, \ldots, n-(1+M)$

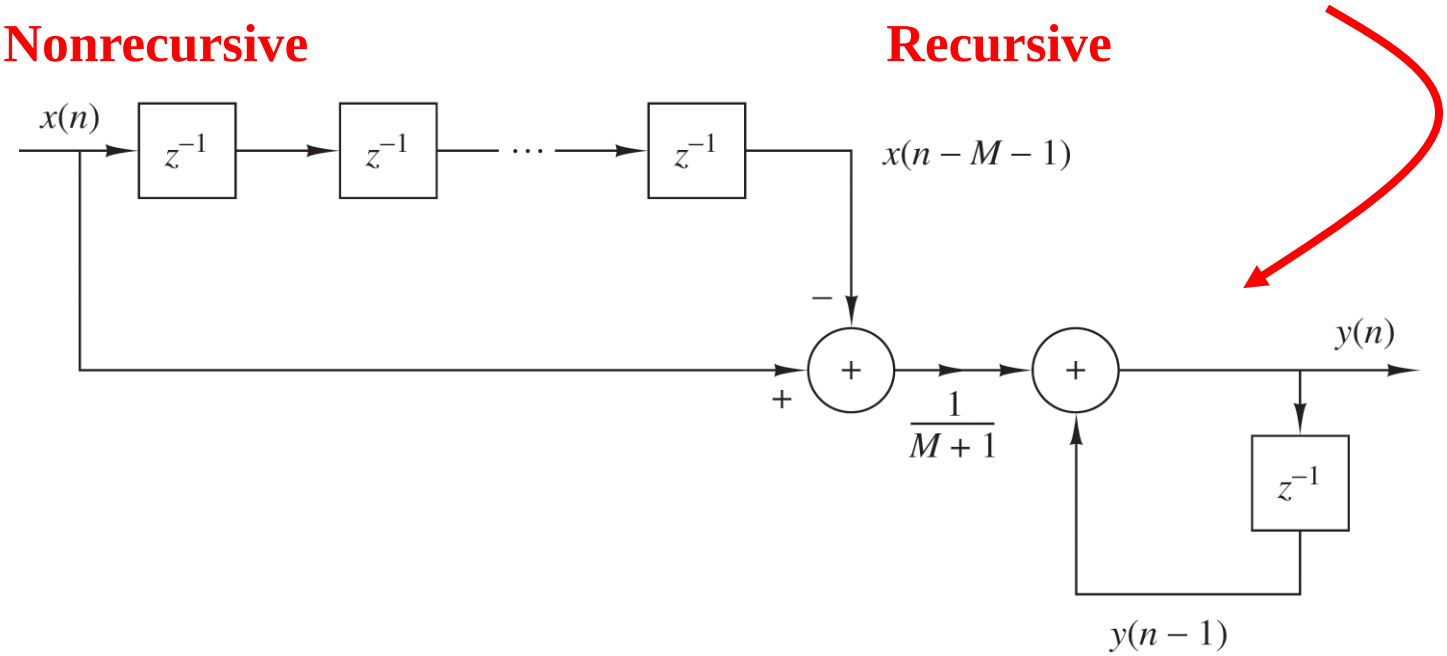
This is actually $y[n-1]$

$$y[n] = y[n-1] + \frac{1}{M+1}\left(x[n] - x[n-(1+M)]\right)$$

Recursive!

$$y[n]=\frac{1}{M+1}\sum_{k=0}^{M}x[n-k] \quad = \quad y[n]=y[n-1]+\frac{1}{M+1}\left(x[n]-x[n-(1+M)]\right)$$

Nonrecursive **Recursive**



Summarzing:

- FIR & IIR describe fundamental characteristic of the system's impulse response regardless of how it is implemented
- Recursive and Non-Recursive describe the specific structure used to implement the system

Nonrecursive → FIR

Recursive → IIR or FIR

Discrete-Time System Relationships